Identification and propagation of parametric uncertainty of a hydrostatic drive train model

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Abstract

This paper shows an approach to identify the uncertainty of the parameters in a lumped parameter model from experimental data. This approach is applied to a hydrostatic drivetrain setup in which the behavior of both the hydraulic pump and motor vary as the temperature of the oil changes. The model contains a large amount of parameters, describing the dependency between the speed, pressure and stroke of pump and hydromotor and their efficiency. To decide which parameters to retain for the uncertainty identification, an identifiability study is performed followed by a study of the residuals. The parameter uncertainty identification procedure estimates the parameter standard deviations using a maximum likelihood procedure between the calculated distribution of the model output and the observed residuals. The resulting parameter uncertainty is evaluated in order to propose a design improvement to lower system uncertainty. Also the potential use of the uncertainty model for robust and stochastic control is discussed.

1 Introduction

The last decades physical models are used more and more in the design of machines [1]. These models are used for different design purposes [2] such as conceptual design [3], physical component design [4] and controller design [5]. When analyzing the behaviour of systems, lumped parameter physical models have proven to be a valuable tool because of their limited complexity while remaining sufficiently accurate in many cases [2].

However, a common problem when such a model is used to numerically simulate a real-world system is the fact that the accuracy of a model can be limited due to modeling errors and approximations (i.e. epistemic uncertainty) or due to variability of the modeled components (i.e. aleatory uncertainty) [6]. This causes the estimated values for the parameters of such models to exhibit a high level of uncertainty. Consequently, relying on simulation results obtained using only one specific value for the model parameters might not be an accurate description of the true system behavior. As shown in figure 1, when comparing numerical simulations with experimental testing, there will always be a different between the two, however there is no way of determining if this difference is significant or not. To avoid these, the model uncertainties should be appropriately taken into account in the simulation procedure allowing the computation of the likelihood of the model being a good representation of the true system behavior.

Whenever numeric simulations are used for making design decisions or for robust control design, a quantitative assessment of model uncertainty provides relevant information. Nowadays a lot of research results are available on the propagation of parameter uncertainty/variability [7]. In practice, however, the uncertainty/variability of the model parameters is often unknown. For instance, in the case the parameter has been estimated by fitting model simulations to experimental data, the corresponding parameter uncertainty is to be estimated from the same experimental data. This is referred to as uncertainty identification.

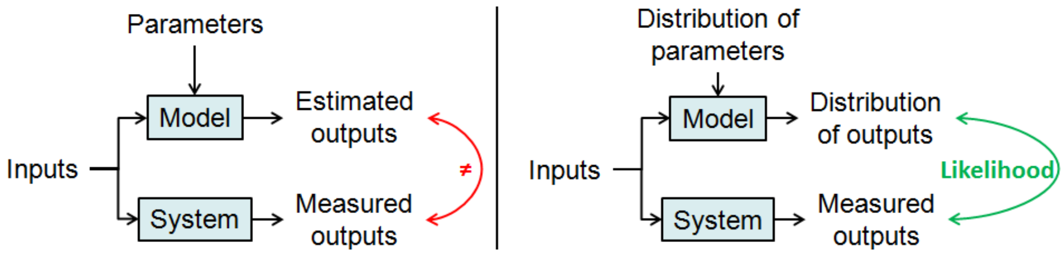

Figure 1: The advantage of uncertainty modeling.
Left: Comparing a deterministic model with measurements gives no indication of how good the model is.
Right: Comparing an uncertain model with measurements, gives a likelihood of the model being correct.

This paper presents an approach to perform uncertainty identification, and shows its application on a lumped-parameter model describing the steady-state behavior of a hydrostatic drive-train experimental setup. On this setup, mechanical power is provided by an electric motor. This mechanical power is converted into hydraulic power by a variable-displacement pump. The hydraulic power is then converted back into mechanical power by a variable-displacement hydromotor to drive a load, on our setup simulated by an electro motor. The transmission ratio of the hydrostat is determined by the ratio of the displacement volumes (or strokes) of pump and hydromotor. Since the behavior and efficiency of both the hydraulic pump and the hydromotor vary greatly as the temperature of the oil changes [9], a lumped-parameter model of this system has to either include a thermal model, or take into account this variable behavior by adding uncertainty to the parameters in the model. As the derivation of a thermal model requires additional sensors and a large amount of experiments, this second approach was chosen for this paper.

To identify both the mean value of the parameters as well as their uncertainty, experiments have been performed in a number of steady state operating points, covering as much of the operational area (speeds, torques and displacements) as possible. Using these measurements, a suitable 1D-lumped parameter physical model was created and fitted, describing the system response and energy losses in steady state. This model contains a large amount of parameters, describing the dependency between the speed, pressure and stroke of pump and hydromotor and their volumetric and mechanical efficiency. An identifiability study revealed that there is a significant correlation between several parameters, making it impossible to quantify the uncertainty for all of them. To discern which parameters are the most likely candidates to explain the residuals, the differences between the model and the experiments, a residual study is performed. This study determines the correlation between the observed residuals and the distribution of the model output caused by propagated parameters uncertainties. Only those parameters with the highest correlation are retained for the uncertainty identification. The parameter uncertainty identification procedure itself assumes a normal distribution for each retained parameter and estimates their standard deviation. The model is linearized around its fitted parameter values, allowing simple affine transformation rules to be used to compute the standard deviation of the model output, based on the standard deviation of the parameters. An optimization algorithm then searches for the parameter uncertainties that result in the highest likelihood of the observed residuals. The resulting parameter uncertainty is evaluated in order to propose a design improvement to lower system uncertainty. Also the potential use of the uncertainty model for stochastic control is discussed.

As shown in figure 2, in sections 2 to 6 we detail the followed approach. In section 7 we show how the identified uncertainty has been applied to determine a good design improvement. Sections 8 and 9 give conclusions and planned further work on robust controller design.

2 Hydrostatic drivetrain model

Figure 3 shows the scheme of the main components in the experimental setup. A lumped-parameter model is created describing the steady-state behavior and the energy losses in the hydrostatic drivetrain. The driving

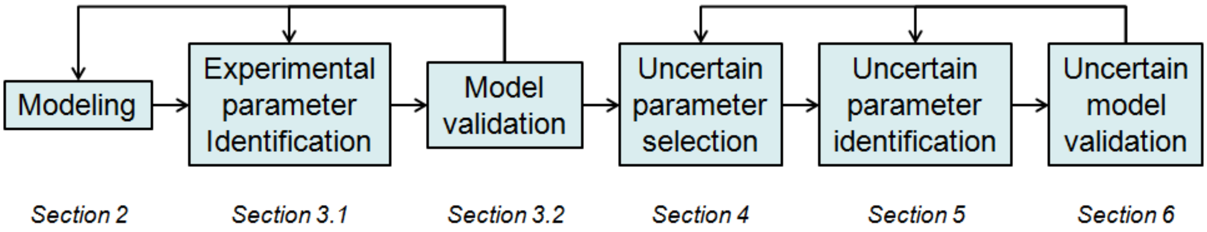

Figure 2: The different steps of the approach and the section in which each step is described

electric motor is speed controlled, fixing the rotational speed of the pump shaft while the load motor is torque controlled, fixing the torque flow that has to be delivered by the hydromotor. The pump and hydromotor are modeled respectively as a conversion from mechanical power to hydraulic and from hydraulic to mechanical power. Both these hydraulic components have a variable displacement volume.

The energy losses in these hydraulic components are described using two efficiencies, namely the mechanical and the volumetric efficiency. The mechanical efficiency is mainly caused by friction, whereas the volumetric efficiency is caused by leak flow. According to [8], both of these losses are dependent on (i) the speed of the mechanical shaft, (ii) the pressure difference over the hydraulic ports and (iii) the displacement volume They can be modelled according to the following formulas:

$$T_{fric} = D \times p \times k_0 \times \left(\frac{p}{p_{Nom}}\right)^{k_p} \times \left(\frac{\omega}{\omega_{Nom}}\right)^{k_\omega} \times \left(\frac{D}{D_{Nom}}\right)^{k_D} \tag{1a}$$

$$q_{leak} = D \times \omega \times l_0 \times \left(\frac{p}{p_{Nom}}\right)^{l_p} \times \left(\frac{\omega}{\omega_{Nom}}\right)^{l_\omega} \times \left(\frac{D}{D_{Nom}}\right)^{l_D} \tag{1b}$$

D : displacement volume

p : the pressure over the hydraulic ports

ω : shaft speed

These emprical equations contain eight parameters to describe the losses per hydraulic component. l_0 and k_0 model the losses at the nominal operating point of the hydraulic component, while k_p, l_p, k_ω, l_ω, k_D and l_D indicate the dependency of the friction torque and leak flow on the relative pressure, speed and displacement. Figure 4 shows the typical shape of the resulting mechanical efficiency as a function of the speed and pressure, using formula 1a for the friction torque.

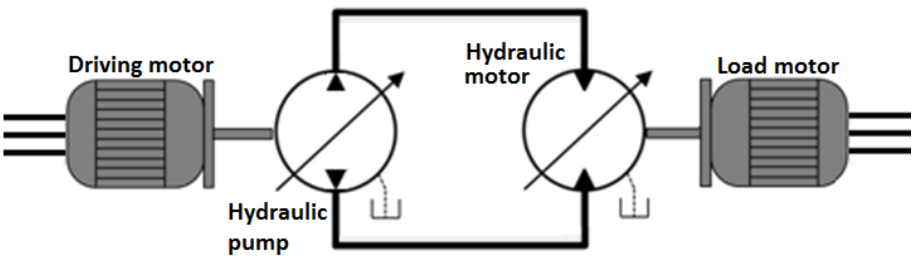

Figure 3: Schematic of the experimental setup

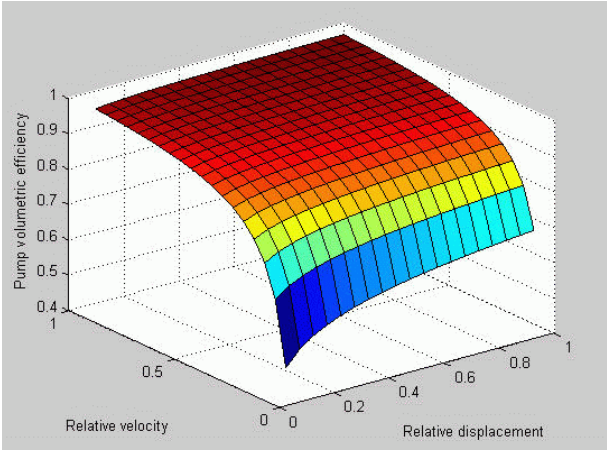

Figure 4: Typical shape of the pump mechanical efficiency as a function of the relative speed and the relative displacement

3 Experiments

3.1 Full-factorial design

In order to identify the hydraulic pump and motor, a number of experiments were performed. Sensors were placed on both mechanical shafts, measuring their speed and the torque flowing through them, but no sensors are present in the hydraulic circuit. This way the total mechanical efficiency, caused by friction, and volumetric efficiency, caused by leak flow, of the pump and hydromotor can be measured, however the lack of hydraulic sensors prevents separating the efficiencies of pump and hydromotor.

As described in the previous section, the efficiency of the variable-displacement hydraulic components in the system depends on (i) the speed of the mechanical shaft, (ii) the pressure difference over the hydraulic ports and (iii) the displacement volume. This was taken into account by the design of experiments, which ensured that the operational area of each of these three influences was explored using a so called full factorial design [10]. However, since the pump behavior doesn't depend on the hydromotor stroke and vice versa the pump and hydromotor strokes were not varied relative to each other. This led to two sets of steady-state measurements. The first set varied the speed of the driving motor, the stroke of the pump and the torque of the load motor, while keeping the hydromotor stroke constant. In the second set of measurements the pump stroke is kept fixed while the hydromotor stroke is varied. For both measurement sets, a full factorial of all three inputs being varied was used, as shown in figure 5, this resulted in 1440 measurements per set $(k \cdot m \cdot n = 1440)$

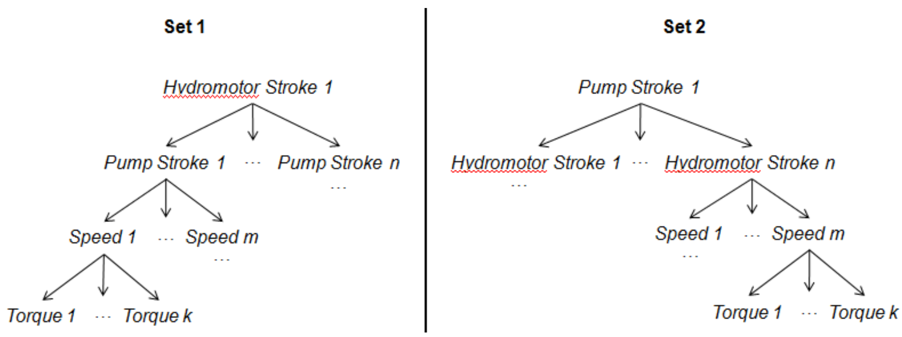

Figure 5: Resulting design of experiments using full factorial

3.2 Model update

By using equations (2) and (3), the measured torques and speeds were used to determine the total mechanical and volumetric efficiency of the hydrostatic drivetrain. This resulted in the volumetric efficiency measurements shown in figure 6. These measurements contain efficiencies above 100%, which cannot be explained with the current model. Ordering these measurements by the different inputs being controlled (driving motor speed, load motor torque, pump and hydromotor displacement volume), revealed that these abnormal efficiencies are present at low pump strokes. This phenomenon can be explained using equation (3) and the pump spec sheet.

$$\eta_{Mech,Total} = \frac{T_{Motor,\,Measured} \times D_{Pump,\,Actual}}{T_{Pump,\,Measured} \times D_{Motor,\,Actual}} \tag{2}$$

$$\eta_{Vol,Total} = \frac{\omega_{Motor,\,Measured} \times D_{Motor,\,Actual}}{\omega_{Pump,\,Measured} \times D_{Pump,\,Actual}} \tag{3}$$

The spec sheet of the pump showed that the actual displacement volume of the pump can deviate from the desired one by up to 5% of the maximal displacement volume. So at low pump strokes, using the desired stroke instead of the actual stroke to compute the volumetric efficiency can cause the values above 100%. To solve this, an additional parameter was added to the model, namely a pump stroke deviation Δ. Equation (4) was added to compute the actual pump stroke based on the desired stroke.

$$D_{Pump,\,Actual} = D_{Pump,\,Desired} + \Delta \tag{4}$$

It was decided not to add such a parameter to the hydromotor stroke since (i) the spec sheet showed that this effect was less prominent in the hydromotor and (ii) the hydromotor was mostly used at high strokes, where this deviation has much less impact The resulting model fit after the model update is shown in figure 7.

4 Variable parameter selection

The model built up in the previous sections contains a total of 17 parameters. In order to estimate the variability of each of these parameters from a set of experiments on the complete system requires the effect each of them has on the system response (the total mechanical and volumetric efficiency) to be signifcantly different.

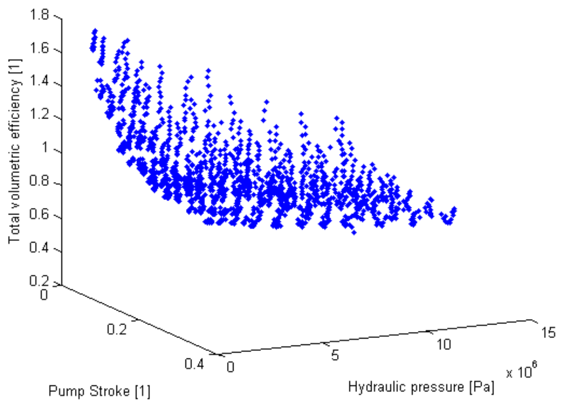

Figure 6: Measured total volumtric efficiency as a function of the pump stroke and pressure

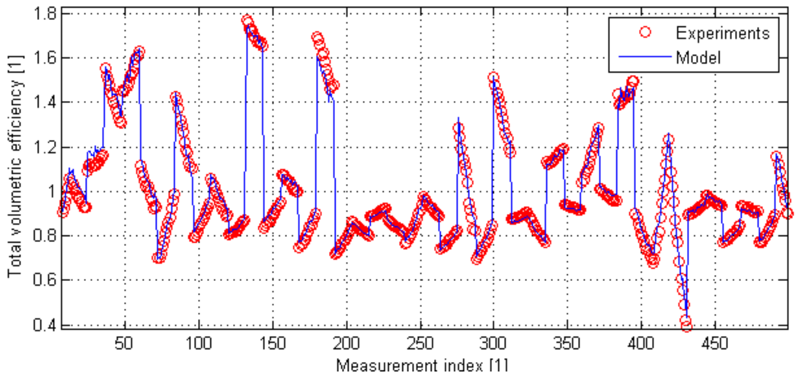

Figure 7: Measured and simulated volumetric efficiency for a number of measurements

4.1 Identifiability study

In order to detect if each model parameter has a significantly different effect on the model outputs (total mechanical and volumetric efficiency), an identifiability study [11] is performed. This study investigates how much the impact of a parameter on the model outputs can be replicated by another parameter. To quantify this, the dependency between two parameters p_i and p_j can be determined in three steps:

1. Determine the fitting error E_0 between the model and the measurements at the identified optimal parameter values.
2. Fix parameter p_i at a non-optimal value, and determine the new increased fitting error E_1 .
3. Optimize the value of p_j to minimize the fitting error E_2 with the non-optimal for p_i.

The dependency between p_i and p_j is then given by

$$Dependency = \frac{E_1 - E_2}{E_1 - E_0}. \tag{5}$$

If this dependency equals 1, then the impact of p_i on the system output can be completely replicated by changing p_j. A value of 0 indicates a complete independence between p_i and p_j. The result of the identifiability study on the hydrostatic model are visualized in figure 8, resulting in the following observations:

- The parameters of a single efficiency map are dependent on eachother, shown by the brighter blocks around the diagonal. This makes it difficult to distinguish between their impact on the total mechanical and volumetric efficiency,
- The parameters of the hydromotor efficiencies show less dependency. We assume this is caused by speed limitations in the system that prevented the full exploration of the [0,1] interval of the relative speed, pressure and stroke of the pump, while not for the hydromotor. This makes it easier to distinguish the effect of the different parameters of the hydromotor efficiencies on the system output.
- There is somewhat of a dependence between the pump stroke deviation and the parameters of the volumetric efficiencies. We assume this is because both of them determine the hydraulic flow.

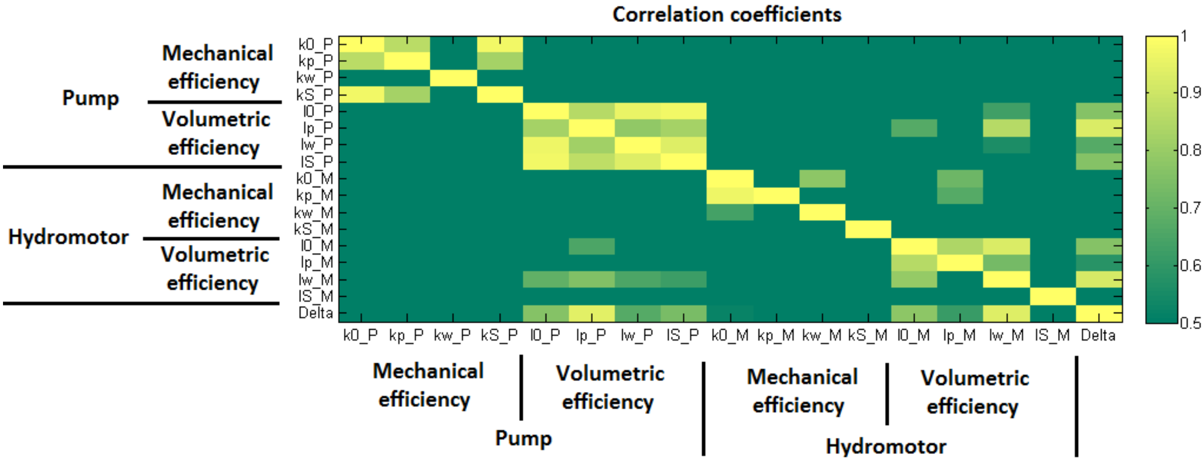

Figure 8: Visualization of the 1-on-1 dependency between the different parameters. Dark green indicates little to no dependency. Brighter colors indicate a stronger dependency.

4.2 Residual study

The identifiability study revealed that there are several groups of dependent parameters in the model. Mostly the parameters per efficiency map are dependent on one another, making it difficult to seperately estimate the variability of each parameter. Since we are mostly interested in the global variability on the efficiency maps, it was decided to reduce the number of variable parameters per component efficiency map (mechanical/volumetric of pump/hydromotor) to one. In order to choose which parameters are classified as uncertain, a residual study is performed [12].

The residual study performed loops over all 17 parameters and determines per parameter p_i how well the observed residuals (δ_j *with* $j = 1..n_{Meas}$) can be explained by assuming only this single parameter as uncertain. This is achieved by computing the standard deviation of each parameter that results in the highest likelihood of the observed residuals. This likelihood equals the probability of each of the residuals, given by equation (6). This equation assumes a zero-mean normal distribution for the output with a known standard deviation. The standard deviation of output j, σ_{η_j}, is approximated according to (7), with σ_{p_i} the standard deviation of the uncertain parameter and $\frac{\partial \eta_j}{\partial x}$ the jacobian of output j with respect to this parameter.

$$p(\delta_j) = \frac{1}{\sigma_{\eta_j}\sqrt{2\pi}} e^{-(\delta_j)^2 \big/ 2\sigma_{\eta_j}^2} \tag{6}$$

$$\sigma_{\eta_j} = \left| \frac{\partial \eta_j}{\partial p_i} \right| \times \sigma_{p_i} \quad \forall j = 1..n_{Meas} \tag{7}$$

The results of this resiual study are shown in table 1. This table shows the maximal logarithm of the likelihood of each parameter, ordered per component and effiency map. Per identified group of strongly dependent parameters, the parameter with the highest value is kept as variable, namely $k_{p,Pump}$, $k_{p,Motor}$, $l_{p,Pump}$, $l_{0,Motor}$ and Δ.

5 Variability identification

In order to identify the uncertainty of the different model parameters resulting from the variable behavior of the hydrostatic drive-train, a maximum likelihood method is used. This approach start from two assumptions:

	η_{Mech}				η_{Vol}			
	k_0	k_p	k_ω	k_S	l_0	l_p	l_ω	l_S
Pump	$-8 \cdot 10^4$	$\mathbf{4 \cdot 10^3}$	$-9 \cdot 10^9$	$-3 \cdot 10^4$	$5 \cdot 10^3$	$\mathbf{9 \cdot 10^3}$	$6 \cdot 10^3$	$6 \cdot 10^3$
Motor	$-5 \cdot 10^7$	$\mathbf{-3 \cdot 10^6}$	$-1 \cdot 10^9$	$-2 \cdot 10^{11}$	$\mathbf{9 \cdot 10^3}$	$-9 \cdot 10^5$	$-5 \cdot 10^4$	$7 \cdot 10^3$
$\mathbf{\Delta}$	$1 \cdot 10^4$							

Table 1: Maximal achievable log(likelihood) of the residuals when using just one variable parameter

1. the parameters are uncorrelated and have a normal distribution
2. a linearized model is sufficiently accurate within the resulting output confidence intervals

When both assmption hold, equation (8) can be used to compute the standard deviation on the mechanical and volumetric efficiencies σ_{η_j} for given parameter standard deviations σ_{p_i}.

$$\sigma_{\eta_j} = \sqrt{\sum_i \left(\frac{\partial \eta_j}{\partial p_i}\right)^2 \times \sigma_{p_i}^2} \quad \forall j = 1..n_{Meas} \tag{8}$$

The maximum likelihood algorithm will determine the value of the σ_{x_i}'s that maximizes (9), the likelihood of the observed residues δ_j.

$$\prod_{j=1}^{n_{Meas}} p(\delta_j) = \prod_{j=1}^{n_{Meas}} \frac{1}{\sigma_{\eta_j} \cdot \sqrt{2 \cdot \pi}} e^{-(\delta_j)^2 / 2 \cdot \sigma_{\eta_j}^2} \tag{9}$$

For numerical reasons, the objective function is defined as the logarithm of (9). The optimization uses a gradient-based solver and results in the coefficients of variation in table 2.

	Parameter	c_v [%]
Mechanical efficiency	$k_{p,Pump}$	8.35
	$k_{p,Motor}$	7.30
Volumetric efficiency	$l_{p,Pump}$	0.00
	$l_{0,Motor}$	0.00
	Δ	11.55

Table 2: Identified coefficients of variation, classified per efficiency they impact the most

This table shows that the fitting error on the volumetric efficiency was completely ascribed to the uncertainty on the pump stroke deviation. The variability of the mechanical efficiency seems to be caused by a change in the pressure depence, which is modelled by $k_{p,Pump}$ and $k_{p,Motor}$.

6 Variability propagation & validation

In order to check the computed variabilities, a validation experiment is performed. In this experiment a small set of steady-state measurements is performed several times in a row, resulting in an increased temperature every time the set is repeated.

The inputs used for these measurements are also applied to the model, and through equation (8) the output variability is computed. Figure 9 show the results, in which 75% of the measurements lay within the 66% confidence interval. We assume that this slight overestimation of the output variability is because of the linear approximation that was applied to a non-linear system.

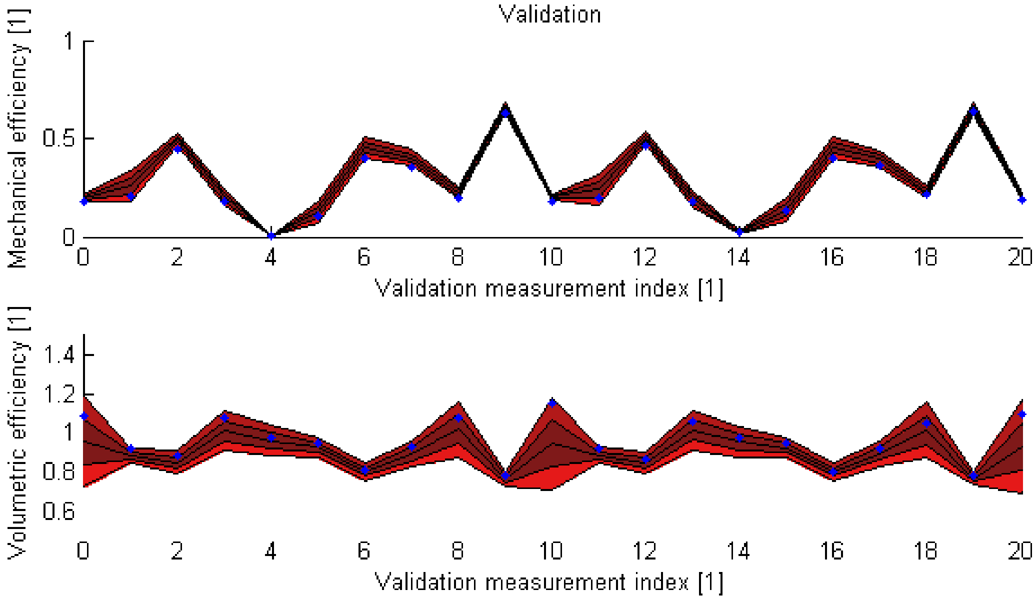

Figure 9: Validation results for a number of measurements.
The dark and light red bands are the 66% and the 95% confidence intervals respectively.

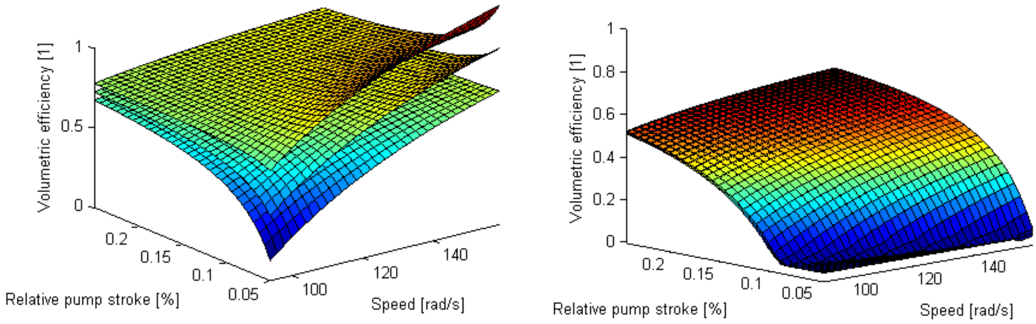

Figure 10: Impact of adding a pump stroke feedback controller on the volumetric efficiency uncertainty
Left: without system upgrade, Right: with system uprade

7 Application: design improvement

The current system model has an unpredictable behavior at low pump stroke. This behavior is caused by the pump stroke deviation parameter and its uncertainty, which causes a deformation of the efficiency maps, along with large confidence bounds.

In order to improve the system response and reduce the size of these confidence intervals a feedback loop can be added to the pump stroke. This loop would guide the actual pump stroke to the desired stroke, greatly reducing the uncertainty on the actual pump stroke, thereby removing the need for the deviation parameter. Such a feedback loop requires adding a sensor to the pump that measures the actual pump stroke. Since the accuracy of such a sensor is limited, the uncertainty on the pump stroke is reduced but not completely removed. Figure 10 shows the impact this system upgrade can have on the nominal volumetric efficiency and its confidence bounds, left without the feedback loop, right with a feedback loop. In this figure the sensor and loop remove the nominal deviation at low pump strokes and reduce the uncertainty on the pump stroke by 90%, resulting in an overall reduction of the uncertainty on the volumetric efficiency of 90%.

8 Conclusion

This paper has detailed an approach to identify the variability in a hydrostatic drive-train. This was achieved by identifying the uncertainty on the different parameters in a lumped parameter physical model of the system. The approach started by building a physical model, experimentaly identifying the nominal value of the different parameters and validation the system response. This validation showed the need for a model upgrade. The resulting model contained a large number of parameters, some of which were highly correlated, as shown in an identifiability study. To select which parameters are assigned an uncertainty, an analysis of the residues was performed. A maximum likelihood optimization was then performed to identify the uncertainty on each of the selected parameters. Finally, the resulting uncertainties were experimentally validated. The application of the identified uncertainties in determining the advantage of a design improvement was described.

9 Further work: robust & stochastic control

For a given desired load speed and torque, the hydrostatic drive-train has a number of degrees of freedom. Indeed, one particular transmission ratio can be produced by multiple pump and hydromotor stroke combinations. On top of that, multiple combinations of transmission ratio and driving motor speed yield the same output speed and torque. So there are at least two redudant degrees of freedom. Those can be exploited to optimize a certain criterion, such as energy efficiency, i.e. minimize the overall losses Hydrostatic drive-trains are typically used in heavy off-road vehicles, where the driving motor is an internal combustion engine. Reducing fuel consumption for these machines is a major motivation for model-based optimal control.

For calculating the optimal inputs of the drive-train, we have to rely on the parametric model. If we do not take the uncertainty into account that arises from the varying oil temperature and other effects, the calculated operating point might be infeasible or suboptimal in practice. For example, the settings might yield a too high relative pressure resulting in a bypass flow through a pressure relief valve. When propagating the uncertainty on the parameters to the losses and constrained variables, an operating point can be calculated that remains on the safe side with respect to the constraints, while minimizing the average expected losses. A validation of this on the experimental set-up is work in progress.

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