Abstract—This paper discusses the $H_\infty$ based design of a vibration controller for an industrial pick-and-place machine. A vibration controller is added to the classical motion control scheme with the purpose of improving positioning behavior by reducing the vibration level and settling-time. It is shown that a trade-off is required between vibration reduction and motion control. The approach is validated experimentally and the results clearly illustrate the benefit of the proposed method.

I. INTRODUCTION

High-speed, high-accuracy operation and low-cost construction are the main thrusts behind the design of light-weight robotic manipulators, pick-and-place machines, etc. Light-weight structures operating at high speeds may suffer significant vibration problems, thus degrading positioning accuracy and requiring larger settling times. Furthermore, the dynamic behavior of a machine tool can depend on the position of the tool in its workspace. High-performance motion controllers that take into account these varying dynamics are needed.

This paper discusses the design of controllers to enable fast point-to-point movements for a pick-and-place machine with position dependent dynamics. The classical motion control scheme is extended with a vibration controller, in order to improve positioning performance, by reducing the vibration level and settling time. This approach requires a trade-off between vibration and motion control. The design is explained for fixed operating points and the position dependent dynamics can be taken into account using gain scheduling [9].

The implementation of a gain scheduling technique for the considered setup is addressed in [1].

The outline of this paper is as follows. Section II discusses the experimental setup and the identification of linear models. The control structure and the approach to improve positioning are explained in section III. Section IV presents the design of a vibration controller. Section V explains the design of a motion controller. Section VI discusses the experimental results and the trade-off between vibration reduction and motion control. Finally, conclusions and future work are presented.

II. EXPERIMENTAL SETUP

A. Description of the setup

The considered test-case is an industrial 3-axis pick-and-place machine shown in Fig. 1. The Y-motion is gantry driven by two linear motors (1) and the X-motion of the carriage over the gantry is also driven by a linear motor (2). The vertical Z-motion is actuated by a rotary brushless DC-motor which drives a vertical beam by a ball screw/nut combination (3). The position of the linear motors and the length of the beam are measured with optical encoders. In addition, the acceleration in the X-direction of the end point of the beam (4), is measured with an accelerometer.

The objective is to position the end point of the beam as accurately and quickly as possible. However, fast movements of the linear motors excite the eigenfrequencies of the flexible beam and during motion, the length of the beam can continuously change, giving rise to varying resonance frequencies. This paper focuses on motion and vibration control in the X-direction for fixed values of the length of the beam in the Z-direction.
B. Identification of linear models

The setup is identified for different beam lengths. For each length, two frequency response functions (FRFs) are measured using multi-sine excitation [10]. The X-motor has one input, the applied force, and two outputs, namely the motor position and the acceleration of the end point of the beam, which is a non-collocated measurement. The FRF from motor force to motor position for different beam lengths is shown in Fig. 2. A mass-line characteristic (-40 dB/decade) can be recognized in the mid-frequency range. For lower frequencies, the slope tends to -20 dB/decade due to the presence of viscous and hysteretic friction [12]. For higher frequencies, the FRF exhibits a varying resonance, which is the first eigenfrequency of the beam. The FRF from motor force to end point acceleration for different beam lengths, is shown in Fig. 3. For frequencies below the eigenfrequency of the beam, the acceleration of the end point is approximately the same as the motor acceleration. Above this eigenfrequency, the acceleration of the end point of the beam lags behind the acceleration of the motor.

Linear models are estimated for the measured FRFs. A model of order 8 is used for the FRF from motor force to motor position:

\[
G_{\text{POS}} = \frac{K}{m \cdot s^2 + c \cdot s} \cdot \prod_{i=1}^{3} \left( \frac{\omega_{1,i}^2}{\omega_{1,i}^2} + \frac{2 \xi_{1,i} \cdot s}{\omega_{1,i}} + 1 \right) \cdot \prod_{j=1}^{3} \left( \frac{\omega_{2,j}^2}{\omega_{2,j}^2} + \frac{2 \xi_{2,j} \cdot s}{\omega_{2,j}} + 1 \right).
\]  

(1)

The first part of the expression accounts for the low-frequency behavior of the motor and the other second order factors in the numerator and denominator account for the three antiresonance/resonance pairs seen in Fig. 2. The lowest antiresonance/resonance pair depends on the length of the beam and the two pairs at higher frequencies account for the effect of machine frame resonances.

A model of order 4 is used for the FRF from motor force to end point acceleration:

\[
G_{\text{ACC}} = \frac{K \cdot s^2}{(m \cdot s^2 + c \cdot s) \cdot \left( \frac{s^2}{\omega_{1,1}^2} + \frac{2 \xi_{1,1} \cdot s}{\omega_{1,1}} + 1 \right)}.
\]  

(2)

For low frequencies, this model coincides with the model relating motor force to motor acceleration and the additional term of order two in the denominator takes into account the eigenfrequency of the beam. For each FRF and each beam length, the parameters \( K, m, c, \omega_{b,j} \) and \( \xi_{b,j} \) are estimated using a nonlinear least squares algorithm [5]. The details of this procedure are omitted.

III. CONTROL STRUCTURE

The discussion in the following sections focuses on the design of controllers for one fixed beam length. The same procedure can be readily applied
for other beam lengths. Subsection III-A discusses the control structure to improve positioning and subsection III-B explains the design approach.

A. Control structure

As shown by previous studies [11][7], an appropriate way of combining vibration control with motion control is to use a HAC-LAC structure [8] as depicted in Fig. 4. A HAC-LAC structure consists of a high-authority motion controller $C$ (HAC), built around the system with a low-authority vibration controller $D$ (LAC). The purpose of the vibration controller is to assure that the acceleration of the end point of the beam $ACC$ tracks a reference acceleration $R_{ACC}$, which is taken to be zero, but can also be used for acceleration feedforward. The motion controller $C$ is a lead-lag controller [3], used to control the position of the motor $POS$ and to track a reference position $R_{POS}$.

B. Design approach

In the case considered in [11], the influence of the varying eigenfrequency of the beam on the FRF from motor force to motor position was quite negligible, allowing for a decoupled design of the motion and vibration controller. In this paper, the influence of the varying resonance is clearly not negligible, as seen in Fig. 2. Consequently, the design of the HAC-LAC structure is carried out as a two-stage procedure, based on the modeled FRFs $G_{ACC}$ and $G_{POS}$. These models may be different from the actual FRFs $G_{ACC}$ and $G_{POS}$, as indicated in Fig. 4, either due to model mismatches or slight variations of the length of the beam. Firstly, only the control scheme within the dash-dotted rectangle in Fig. 4 is considered and the vibration controller $D$ is designed, while the motion controller $C$ is omitted in this stage. The input of this closed loop system is the reference acceleration $R_{ACC}$ for the end point of the beam. The outputs are the motor position and the acceleration of the end point.

Secondly, the motion controller $C$ is designed around the system contained within the dash-dotted rectangle in Fig. 4, namely the original system, augmented with the vibration controller $D$. The design of the vibration and the motion controller is discussed in section IV and section V respectively.

IV. DESIGN OF THE VIBRATION CONTROLLER

A. Design considerations

The vibration controller needs to be active only in a frequency range around the eigenfrequency of the beam, to minimize interference with the motion controller, since a HAC-LAC structure is used and also because the undesired vibrations of the end point are predominantly present in this range. Additionally, to limit noise sensitivity and prevent actuator saturation, the control action of the vibration controller should be small at frequencies well above the eigenfrequency, and should also be small below this frequency, since accelerometer measurements are generally inaccurate for low frequencies. Around the eigenfrequency of the beam, the end point acceleration $ACC$ needs to track the reference acceleration $R_{ACC}$ as closely as possible and good disturbance rejection is desired, implying that the sensitivity $S = \frac{1}{1 + D \cdot G_{ACC}}$ should be as small as possible [4]. This in turn implies that the complementary sensitivity $T = \frac{D \cdot G_{ACC}}{1 + D \cdot G_{ACC}}$ is around one (0 dB) in this frequency range, since $S + T = 1$. However, this range can not be made arbitrarily narrow, as this leads to peaking of the sensitivity $S$ [4], which is undesirable, as expressed by Doyle’s stability robustness criterion [2]:

$$\left| \frac{G_{ACC}(j\omega) - \tilde{G}_{ACC}(j\omega)}{\tilde{G}_{ACC}(j\omega)} \right| < \frac{1}{S(j\omega)}, \quad (3)$$

Here, $S$ is the sensitivity calculated from the modeled transfer function $G_{ACC}$ and a given controller $D$ and $\tilde{G}_{ACC}$ is the actual transfer function. This inequality expresses that stability for the closed-loop system is only guaranteed for relative mismatches of $G_{ACC}$ that are smaller than $\frac{1}{S(j\omega)}$.

A certain degree of robustness is required, especially if the controller is used in conjunction
with gain scheduling. In linearization gain scheduling [9], controllers are designed at a fixed operating point, but are used in a range around this design operating point. For this reason, performance robustness is equally desirable. For a given vibration controller $D$, the following equality holds [4]:

$$
\frac{T(j\omega) - \tilde{T}(j\omega)}{T(j\omega)} = S(j\omega) \cdot \frac{G_{ACC}(j\omega) - \hat{G}_{ACC}(j\omega)}{G_{ACC}(j\omega)},
$$

(4)

where $\tilde{T}$ is the complementary sensitivity calculated from the actual transfer function $\hat{G}_{ACC}$. This equation expresses that the complementary sensitivity, representing the closed loop performance, will vary little as a consequence of relative mismatches of $G_{ACC}$, wherever $S$ is small. As a result, stability robustness and performance robustness are obtained by making $S$ small. However, to limit interference with the motion controller, the frequency range in which this is realized, can not be too wide.

**B. $H_\infty$ design of the vibration controller**

A vibration controller $D$ is determined for a beam length of 0.15 m, using an $H_\infty$ based approach by solving the mixed sensitivity problem [13][4]. In the SISO case, this problem can be solved by finding the controller $D$ that minimizes [4]:

$$
\sup_{\omega \in \mathbb{R}}(|W_1(j\omega)S(j\omega)|^2 + |W_2(j\omega)T(j\omega)|^2),
$$

(5)

with $W_1$ and $W_2$ suitably chosen, stable, rational functions of $j\omega$. If the minimum value of (5) equals $\gamma^2$, then the optimal solution satisfies:

$$
|S(j\omega)| \leq \frac{\gamma}{|W_1(j\omega)|}, \quad \omega \in \mathbb{R},
$$

(6)

$$
|T(j\omega)| \leq \frac{\gamma}{|W_2(j\omega)|}, \quad \omega \in \mathbb{R}.
$$

(7)

Suppose that the eigenfrequency of the beam is situated at $1/\tau_0$. The right hand side of (7) presents an upper bound for $T$. To minimize interference with the motion controller, $T$ needs to be smaller than one outside the frequency range of the eigenfrequency of the beam. To this end, $1/W_2$ can be chosen as a symmetric transfer function of order $2n$ with a maximum amplitude at $1/\tau_0$:

$$
\frac{1}{W_2} = \frac{\alpha \cdot \prod_{j=1}^{2n} (\tau_0 \cdot \lambda_j \cdot s + 1) \cdot (\tau_0 / \lambda_j \cdot s + 1)}{\prod_{j=1}^{n} (\tau_0 \cdot j \cdot s + 1) \cdot (\tau_0 / j \cdot s + 1)},
$$

(8)

$\alpha$ is chosen such that $1/W_2$ attains a value of around 0 dB at $1/\tau_0$ and $\lambda_j > \mu_j > 1$. $n$ and $\mu_j$ determine the width of the peak and $\lambda_j$ are chosen to be sufficiently larger than $\mu_j$. The transfer function $1/W_2$ is shown in Fig. 5 on the right, where $n = 4$, $\alpha = 1.5$, $\lambda_i \approx 17$ and $\mu_j \approx 1.05$. Inequality (7) is usually dominant outside the frequency band of the eigenfrequency of the beam and hence the high- and low-frequency shape of $1/W_2$ is most important.

To obtain a small $S$ around the eigenfrequency of the beam, a lower bound $T_1$ on the complementary sensitivity is chosen firstly. As $1/W_2$ is an upper bound for $T$ for all $0 < \gamma \leq 1$, $T_1$ can be determined from (8) by decreasing $\alpha$, decreasing $\mu_j$, increasing the order $n$ or by a combination of the above, allowing for many degrees of freedom to shape $T_1$. From $T_1$, $1/W_1$ may be calculated as follows:

$$
\frac{1}{W_1} = \eta \cdot (1 - T_1),
$$

(9)

where $\eta \geq 1$ is introduced to ensure that the upper bound for $S$ in (6) is not too restrictive. The transfer function $1/W_1$ is shown in Fig. 5, where $1/W_1$ is calculated from (9) with $\eta = 1.1$ and $T_1$ is calculated from (8) with $n = 8$ and $\alpha = 0.7$. Inequality (6) is usually dominant in the frequency range around the eigenfrequency of the beam, hence the shape of $1/W_1$, is important in this range.

Based on these weighting functions, an $H_\infty$ controller is determined using the MATLAB [6] Robust Control Toolbox. In Fig. 5, the resulting sensitivity $S$ and complementary sensitivity $T$ are compared with the inverse of the chosen weighting functions It can be seen that (7) and (6) are satisfied for $\gamma = 1$. 

![Fig. 5. The upper bounds 1/W₁ and 1/W₂, for the sensitivity S and the complementary sensitivity T](image-url)
V. DESIGN OF THE MOTION CONTROLLER

A. Identification of the system augmented with the vibration controller

The FRFs of the system with the vibration controller can be calculated from the transfer functions $G_{POS}$, $G_{ACC}$ and $D$ or can be measured by applying a multi-sine excitation signal to the system within the dash-dotted rectangle in Fig. 4. The input signal $U$ to the motor is the sum of the excitation signal $U_C$ and the control signal of the vibration controller. The reference acceleration $R_{ACC}$ is kept equal to zero. The FRF from $U_C$ to motor position is shown full-line in Fig. 6 on the left and the FRF from $U_C$ to end point acceleration is shown full-line on the right and both are compared to the FRFs without the vibration controller, shown in dash-dotted lines. For the FRF from $U_C$ to the end point acceleration, the vibration controller realizes increased damping of the eigenfrequency and for the FRF from $U_C$ to the motor position, almost complete suppression of the resonance frequency is obtained.

B. Design of the motion controller

A lead-lag controller [3] is first designed for the system without vibration controller, resulting in a bandwidth of 25 Hz and phase margin of $46^\circ$ for the motion control loop. The resulting loop transfer function of the motion control loop is shown in Fig. 7 on the left. The addition of the vibration controller causes an extra phase lag near the eigenfrequency of the beam in the FRF from $U_C$ to the motor position, shown full-line in Fig. 6 and has important consequences for the bandwidth of the motion controller.

To obtain a bandwidth above or near the eigenfrequency of the beam, it is necessary to add a second lead term to the lead-lag controller, to compensate for this phase lag. This results in a reduction of the vibration suppression, as the second lead term mainly compensates for the effect of the vibration controller. Additionally, it also causes the peaks of the high-frequency machine frame resonances to rise above 0 dB, deteriorating the phase margin and resulting in unacceptable excitation of these resonances. Therefore, there is little choice but to select a bandwidth that is lower than the eigenfrequency of the beam. A standard lead-lag controller is sufficient in this case and is designed to yield a bandwidth of 10 Hz and a phase margin of $55^\circ$. The loop transfer function with the vibration controller is shown in Fig. 7 on the right.

VI. EXPERIMENTAL VALIDATION

The performance of the vibration controller is validated experimentally for two cases. In the first case, a high-bandwidth motion controller, a lead-lag controller designed for the system without vibration controller, is used. To evaluate the performance, a step in the reference position $R_{POS}$ is applied to the motion control loop and the acceleration of the end point of the beam is measured both with and without the vibration controller. The acceleration response of the end point is shown in Fig. 8. In the second case, a low-bandwidth motion controller, a lead-lag controller designed for the system with the vibration controller is used and the abovementioned experiment is repeated. The
results are shown in Fig. 9. The high-bandwidth controller excites the eigenfrequency of the beam more, since the maximum amplitude of the acceleration is higher. For the low-bandwidth controller, the settling time is considerably lower when used in combination with a vibration controller. This is not the case for the high-bandwidth motion controller, due to interference of the motion controller with the vibration controller, although there is some improvement in the peak amplitude of the acceleration. However, the settling time is worse for the high-bandwidth controller, due to the phase lag introduced by the vibration controller. Adding an extra lead term to the motion controller can only partially solve this problem and introduces difficulties with machine frame resonances, as explained in the section V. For applications that require many short point-to-point movements, it is particularly useful to incorporate a vibration controller in the control scheme. Although using a vibration controller requires lowering of the bandwidth of the motion controller, nevertheless, faster positioning can be achieved by a reduced settling time of the end point of the beam.

Fig. 8. Acceleration with and without the vibration controller, for a high-bandwidth motion controller.

Fig. 9. Acceleration with and without the vibration controller, for a low-bandwidth motion controller.

VII. CONCLUSIONS AND FUTURE WORK

A pick-and-place machine with structural flexibilities is identified and controlled so as to achieve fast and accurate positioning. Improved positioning behavior can be obtained by incorporating a dedicated vibration controller into the motion control scheme. The controller design consists of two stages. Firstly, a vibration controller is designed, whereby a trade-off is required between reducing interference with the motion controller and achieving robustness and performance. Secondly, a motion controller is tuned for the system augmented with the vibration controller, whereby a more conservative choice of bandwidth of the motion controller is required, due to the phase lag introduced by the vibration controller. Nevertheless, the experimental results indicate the benefit of this approach and the successive design stages illustrate the ease of the overall method. Ongoing and future research focuses on the combination of different fixed point controllers using gain scheduling [1], in order to achieve fast and accurate positioning for a variable length of the beam.

REFERENCES

