Performance comparison of LPV control and interpolating control for a pick-and-place machine with position-dependent dynamics

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Abstract This paper discusses the control of a pick-and-place machine of which the dynamics change significantly with the position of the handling tool in the workspace. Two control design approaches for systems with varying dynamics are implemented and compared. The first one is Linear Parameter Varying (LPV) control, an analytical design approach that guarantees the stability of the closed loop for all possible parameter variations. The second approach is traditional gain-scheduling control, where several controllers designed for fixed operating points are interpolated to construct a global gain-scheduling controller. Traditional interpolating control shows to have high performance for the set-up, without loss of stability, even though this stability cannot be guaranteed analytically. While in literature LPV control is suggested as the more standardized approach compared to the 'pragmatic' traditional interpolating gain-scheduling approach, it is concluded from the analysis presented in this paper that there are still some cumbersome steps in the LPV design procedure that limit its suitability for industrial applications. Therefore, interpolating gain-scheduling control is proposed as the more useful technique for such systems.
1 Introduction

High-speed, high-accuracy and low-cost operation are the main thrusts behind the design of lightweight mechatronic structures such as robot manipulators, pick-and-place machines, etc. Light-weight structures operating at high speeds, however, may lead to significant vibration problems, that degrade the positioning accuracy. An additional problem for such systems is that the dynamic behavior can depend on the position of the handling tool in its workspace, resulting in position-dependent dynamics. If accurate positioning is desired these phenomena have to be dealt with in the control design for the machine. When the position of the handling tool is measured, the changes in dynamic behavior can be modeled exactly in a deterministic way. Previous research showed that for such systems maximal performance can only be achieved if the coefficients of the controller are also explicitly dependent on the instantaneous position of the handling tool [11]. Such control structure, called gain-scheduling (GS) control, will be adopted in this paper. In general terms, gain scheduling is a controller implementation where the controller coefficients are changed according to the current value of scheduling signals, which may be signals external and/or internal to the plant [8]. Two main approaches can be distinguished for GS [4], namely traditional gain-scheduling control and modern gain-scheduling or Linear Parameter Varying (LPV) control. The fundamental difference between both approaches is that stability and performance of the closed-loop system is guaranteed for given variations of the parameter(s) in the synthesis of modern GS controllers, whereas if traditional GS control techniques are used, robust stability and performance of the closed-loop system for variations of the parameter(s) is not guaranteed and therefore has to be analysed afterwards.

For most traditional GS control techniques the control design is split up in two parts [4]. In a first step, local LTI controllers are designed for linearised models of the plant computed at several fixed values of the varying parameter. The second step is the actual gain-scheduling step, where a global parameter-dependent controller is obtained by interpolating between these local LTI controllers. Therefore these techniques are often called interpolating GS control techniques.

The modern GS control design approach is conceptually quite distinct from the traditional GS control approach since it involves the direct synthesis of a controller from an LPV model. The modern GS control approaches typically utilise norm-based performance measures, in particular the $\mathcal{H}_\infty$ norm. These approaches can be categorised further according to whether they use a small-gain approach or a Lyapunov-based approach [4, 8].

In this paper, specific implementations of both these control methods are implemented for an industrial pick-and-place machine and their performance is compared.

Section 2 details the experimental setup that is worked on. Section 3 presents the LPV GS control design and Section 4 details the interpolating GS control design. In Section 5 the experimental results for both GS controllers are compared and discussed. Section 6, finally, formulates the main conclusions of the paper.
2 Description and modelling of the practical setup

2.1 Description of the setup

The so-called Flex-cell is an industrial 3-axis pick-and place machine driven by linear motors (see Figure 1). The Y-motion is gantry driven by two linear motors and the X-motion of the carriage over the gantry is also driven by a linear motor (type LIMMs). The vertical Z-motion is actuated by a rotary brush-less DC-motor (Parvex XS220) that drives a vertical spindle beam using a ball screw/nut combination. At the end-point of the beam a pneumatic gripper is mounted that can clamp components. The position of the linear motors and the length of the beam are measured with optical encoders with a resolution of 1 µm (Heidenhein LIDA 201) and the acceleration of the gripper at the end point of the beam in the X-direction is measured with an accelerometer (PCB-302B03). The length of the beam is measured via an optical rotary encoder (Parvex C4-2000) mounted on the motor shaft. A dSpace 1103 data acquisition system is used to acquire the position and accelerometer signals and to calculate the control signals, i.e. current signals for the amplifiers of the motors, at a sample frequency of 2 kHz. The Matlab/Simulink environment is used to design and implement the different controllers.

Fig. 1 Picture of the Flex-cell.

The final objective is to move a component clamped in the gripper as accurate and fast as possible along a prescribed trajectory in the X-Z plane. Fast movements of the linear motor will excite the eigenfrequencies of the flexible beam and during motion, the length of the beam is continuously changing, giving rise to varying
resonance frequencies. To achieve a high performance for all operating conditions, GS controllers will be designed for the motion in the X-direction that take into account the actual value of the length of the beam. The control of the motion in the Y-direction is not considered in this paper.

2.2 Dynamic model of the setup

A Single Input Multiple Output (SIMO) dynamic LPV model is derived for the setup. The input is the current send to the X-motor while the outputs are the position of the X-motor and the position of the end-point respectively. The second output, the position of the end-point of the beam, is obtained from an acceleration measurement of the end-point by double integration in combination with a high-pass filter with cut-off frequency of 4 Hz.

This LPV model is calculated as a product of the following models (see Figure 2):

1. a grey-box LPV model of the X-motor with beam and gripper having one input and two outputs \((z_1, z_2)\); this model describes the varying resonance frequency of the beam;
2. two LTI models, one for each output of the grey-box LPV model, describing the dynamical interaction between the machine frame and the X-motor with beam and gripper.

This decomposition of the dynamic behaviour of the system into a series product of models is justified, since the frequency range in which the dynamic behaviour of the grey-box LPV model and the two LTI models dominates is not overlapping. The LPV model is estimated using grey-box identification, that is, a physics-based model is available of which the unknown parameters are estimated using system identification techniques [10]. The physics-based model is derived from the equations of motion of a 2-mass-spring-damper system (see Figure 2).

![Fig. 2 Composition of the complete LPV model. The outputs \(x_1\) and \(x_2\) are the motor position and the position of the end-point of the beam respectively.](image)

The first mass, \(m_1\), represents the mass of the moving frame containing the linear motor, the rotational motor and the clamping of the beam. The second mass, \(m_2\), represents mainly the mass of the gripper. The spring between the two masses, with stiffness, \(k\), represents the stiffness of the beam that changes in function of the length.
Performance comparison of LPV control and interpolating control of the beam \( l \). Experimental identification of the setup shows that also \( c_2 \) is varying according to variations of the length of the beam.

![Graphs showing performance comparison]

**Fig. 3** Bode plot of four local SIMO models. Left side: from motor force to motor position; right: from motor force to position of the end-point of the beam.

The two LTI models are identified as fourth-order models using black-box frequency-domain identification techniques [10].

Figure 3 shows the Bode plot of the LPV model for four different lengths of the beam.

### 3 LPV control design and analysis

#### 3.1 Synthesis of an LPV controller for the setup

The LMI Control Toolbox of Matlab [3] (currently integrated in the Robust Control Toolbox) is used for syncretising an LPV controller for the LPV model derived in Section 2.2. For selecting the weighting functions for the LPV control design, a slightly modified version of the four-block \( \mathcal{H}_\infty \)-control approach [12] has been followed. This approach is detailed in [6] and allows to trade-off robustness to force/torque disturbances versus accurateness of reference following for mechatronic motion systems where disturbances act both on the plant input and the plant output. In [6] also the weighting functions that are used for the control design can be found.

Figure 4 shows the FRFs of the resulting discretised LPV controller for five equidistant lengths of the beam.
The experimental results that will be discussed in Section 5 show that the designed LPV controller has a rather conservative performance. The main reason for this conservatism is that no limitations can be imposed in the control design method that is implemented in the LMI control Toolbox of Matlab on the parameter variation besides the range of variation. Algorithms that include bounds on the speed of variation of the parameter(s) are described in literature [9, 13], but are not available in Matlab\(^1\). Therefore, these new and probably less conservative LPV synthesis techniques are not tested.

### 3.2 Analysis of the performance of the closed-loop system

The LPV controller of Section 3.1 is designed such that the stability of the closed-loop system is guaranteed for the worst-case parameter variation. The only constraint that is imposed on this parameter variation is the range of variation. The question arises what this worst-case parameter variation is for the closed-loop system and if this is a realistic variation. To answer this question, the closed-loop system is simulated for a large variety of reference signals. It turns out that the worst-case pa-\(^1\)This is due to a limited community of users. In the Summer-school on LPV modelling and control organised by DISC in 2006, this topic was discussed, concluding that there is no perspective that more advanced modern LPV control techniques will be available in future Matlab toolboxes. Moreover, in general numerical problems arise when there are more than a few varying parameters and/or bounds on the rate of variation of the parameter must be taken into account.
parameter variation corresponds to a block-wave with a frequency equal to the double of the average resonance frequency of the beam, being \( \pm 88\text{Hz} \). If the amplitude of this block-wave is chosen smaller than or equal to the range of the parameter variation where the LPV controller was designed for (140\( \text{mm} \)) the system remains stable. For a slightly higher amplitude \( \pm 5\% \) the closed-loop system becomes unstable. The same occurs when a sinusoidal signal is used instead of a block wave. This particular worst-case parameter variation can be explained intuitively by considering the underlying dynamic system of the LPV model shown in Figure 2. For a constant stiffness of the beam, \( k \), the relation between the force delivered by the spring and the displacement between the two masses, \( \Delta x = x_1 - x_2 \), is linear, \( f_{\text{spring}} = k(l)\Delta x \), which corresponds to a line with a fixed slope \( k \) through the origin in Figure 5.

When this system vibrates at the resonance frequency, each period, kinetic energy is periodically transformed into potential energy in the spring and vice versa. Without external forces, the total energy in the system is decreasing, due to the friction in the system. However, if the stiffness of the spring is varying, positive energy can be continuously delivered to the system if \( \int_{\text{period}} -f_{\text{spring}} d(\Delta x) \) is positive in each period of the vibration of \( \Delta x \). As shown in Figure 5, the maximal amount of energy that can be inserted corresponds to making the stiffness \( k \) low when \( |\Delta x| \) is increasing and making the stiffness \( k \) high if \( |\Delta x| \) is decreasing, so when \( k \) is varying according to a block-wave. This maximal amount of energy is proportional to the difference between the maximum and minimum value of the stiffness. Since on our setup the maximum and minimum value of the stiffness correspond to the minimum and maximum value of the length of the beam respectively, the inserted energy is proportional to the range of the parameter variation. This explains why the performance of the LPV controller increases when the range is decreased, although the speed of variation of the parameter remains unbounded.

On the setup, the bandwidth of the movement of the arm in the Z-direction is only \( 7\text{Hz} \) (\( \ll 88\text{Hz} \)). Therefore, it can be concluded that this worst-case parameter variation is not realistic for our setup.

**Fig. 5** Graphical representation of the energy input into the system (= shaded area) during one period of vibration when the stiffness \( k \) varies according to a block-wave.

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\begin{align*}
\bar{k} &= k(l_{\text{min}}) \\
\underline{k} &= k(l_{\text{max}})
\end{align*}
\]
4 Interpolating gain scheduling control design

As discussed in the introduction, an alternative for LPV GS control design is interpolating GS controller design. Using the modified four-block $H_\infty$-control design procedure [6], MISO LTI $H_\infty$-controllers are designed for 8 different lengths of the beam. For these designs the same weighting functions are used as those for the LPV controller design in Section 3.1. These local controllers are interpolated to obtain an interpolating gain-scheduling (IGS) controller, which is then used to control the system for all possible values of the length of the beam. The interpolation technique used fits pole and zero loci on the poles and zeros of the local models and is described in detail in [7].

Since the interpolation technique can only be used to interpolate SISO models, each MISO controller has been decomposed into two SISO controllers, for each input one controller. As a result two IGS controllers are designed, a motor controller and an end-point controller. Figures 6 and 7 shows that the Bode plots of the IGS controllers (evaluated at different local values of $l$) are fitting very well on the local LTI controllers.

Measurements show that the performance of the IGS controllers is very similar to the performance of the local LTI controllers. As such, it is to be expected that the IGS controller outperforms the LPV controller designed in the previous section. This will be discussed in Section 5. The stability of the closed loop system for all physically possible parameter trajectories is not guaranteed by the IGS design procedure. After designing the IGS controller, it is therefore necessary to check the stability of the closed-loop system. Two techniques have recently been described in literature to check this closed-loop stability [5, 1]. However, such analysis is out of the scope of this paper, but can be found in [6].
5 Comparison of results and discussion

In this section the experimental results of the IGS controller and the LPV controller are evaluated and compared with those for a lead-lag (LL) controller [2] in order to verify if the advanced $H_{\infty}$-based GS controllers have better performance than a standard LTI controller designed using techniques that are commonly applied in industry.

The LL controller uses only one feedback signal being the motor encoder. As the desired performance specifications for the response of the end-point of the beam are difficult to transform to specifications on the response of the motor, the same controller design specifications cannot be used. Therefore, the LL controller is shaped such that the resulting closed-loop transfer functions have the same basic shapes as the closed-loop transfer functions using the IGS controller.

To analyse the performance of the different controllers typical high-dynamic pick-and-place motion trajectories are applied.

In Figure 8, an overview is given of the settling-times after a step in the reference command of 450 $\mu\text{m}$ for different lengths of the beam. The performance of the IGS controller is clearly the best here and is very constant for different lengths of the beam. The performance of the LPV controller is varying for different lengths of the beam. This is inherent to the design of $H_{\infty}$-based LPV controllers, since only the worst-case performance over the range of operating points is optimised. The performance of the LL controller decreases considerably for higher lengths of the beam, since the natural damping of the resonance of the beam decreases for higher lengths of the beam. For $l = 200\text{mm}$ a comparison of the step-responses using the three control approaches is given in Figure 9.

In Figure 10 the time constants of the exponential decay after an impulse force excitation at the end-point of the beam are plotted for different lengths of the beam. The performance of the IGS controller remains again almost constant for different

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**Fig. 7** Bode plot of the end-point (right) controllers (black lines) and the IGS controller (grey lines) evaluated at 8 different lengths of the beam $l$. 

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**Fig. 8** Settling-times after a step in the reference command of 450 $\mu\text{m}$ for different lengths of the beam.
lengths of the beam. The time-constants of the LL controller and the LPV controller follow more or less the same trend as the time-constants in the case no controller is used (=natural time-constants). For very short lengths of the beam the increase of performance is low, but for large lengths of the beam, the advantage of the IGS controller is clear (improvement of 60% at $l = 220\text{mm}$).

In Figure 11 the response of the end-point of the beam is displayed when an aggressive reference trajectory is applied to both the $X$ and $Z$ motions simultaneously. Also in this experiment the IGS controller outperforms both the LPV controller and the LL controller, although the relative difference between the performance indica-
tor used in this experiment, the $20\mu$ settling-time, is less than in the case of a step in the reference command. The reason for this is that here a trajectory generator is used not to excite the structural resonances.

Fig. 10  Time constants of the exponential decay after an impulse force excitation at the end-point of the beam for different lengths of the beam.

Fig. 11  Response of the end-point of the beam when an aggressive reference trajectory is applied to both the $X$ and $Z$ motions simultaneously.
6 Summary and conclusions

In this paper three different controllers were designed and analysed for the Flex-cell setup: (1) a modern LPV GS controller; (2) a traditional IGS controller and (3) a basic LTI LL controller that only uses the motor encoder as feedback signal. From this analysis it is observed that the performance of the LPV controller is poor compared to the performance of an LTI controller designed for a fixed length of the beam. This poor performance is due to the unrealistic assumptions on parameter variations that are made during the control design. The IGS controller, on the other hand, clearly outperforms the LL and the LPV controller and therefore it can be concluded that for mechatronic motion systems as the one considered in this paper interpolating gain-scheduling control is the more appropriate control technique.

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