Contact Stiffness Characteristics of a Paper-Based Wet Clutch at Different Degradation Levels

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Summary

After clutch engagement in the post-lockup phase, the contact stiffness between friction materials and separators plays an important role in the dynamic behaviour of an Automatic Transmission (AT). The friction material deteriorates progressively during the service-life of a clutch, thus affecting the contact stiffness. The deterioration therefore changes the dynamic behaviour of the AT. In order to be able to predict the dynamic behaviour of the latter in the post-lockup phase, the contact stiffness characteristics at different degradation levels must be investigated. Consequently, this change in the dynamic behaviour can be used as a means to monitor clutch degradation. In this paper, both simple elastic contact model of rough surfaces, and experimental-setup tests are presented. Three identical paper-based friction materials with different degradation levels were used. Disc-on-disc experiments were performed on a newly developed rotational tribometer to simulate the representative post-lockup phase. In the experiments, those identical specimens were immersed in a fresh Automatic Transmission Fluid (ATF). The experimental results qualitatively agree with the presented model. In general, it can be concluded that, due to the friction material degradation, normal contact stiffness exhibits an increasing trend; in contrast, tangential contact stiffness exhibits a decreasing trend.

1. Introduction

In recent years, the use of Automatic Transmissions (AT’s) in automobiles has become increasingly popular, especially in the North American, and Asian markets. In contrast to this fact, AT’s are still not very popular in European market; 80 % of drivers prefer to use a Manual Transmission (MT) [1]. AT’s are driver-friendly and easy to drive, and consequently the demand of AT remains high. Nevertheless, the energy efficiency of AT’s is 5 – 15 % less than that of MT’s [2,3]. In addition, AT’s are also widely used for heavy-duty commercial and industrial vehicles and equipment.

In an AT, wet clutches are commonly used as mechanical elements transferring power from a driving part (e.g. engine) to a driven part (e.g. wheel) through a frictional mechanism. ‘Wet’ here means that the clutches are immersed in an Automatic Transmission Fluid (ATF). The ATF has a function as a cooling lubricant fluid maintaining the surfaces clean and giving smoother performance and longer life. A multi disc wet clutch is the most common configuration, which consists of several friction and separator discs, as shown in Figure 1. In general, the friction discs are mounted to the input shaft by splines, and the separator discs are mounted to the output shaft by lugs. The friction disc is made of a steel disc with a friction material bonded on both sides, and the separator disc is made of plain steel. Moreover, in wet clutch application, a hydraulic actuator is commonly used for engagement and disengagement mechanism. The actuator consists of several main components, such as a hydraulic cylinder and a control valve.

Figure 1: Schematic of a Wet Clutch [4]

Because of a relatively high friction coefficient, stable friction characteristics, and low cost; paper-based friction materials have been popularly used for clutch and brake applications since the late 1950s. This type of
Friction material is also called the high performance paper, which consists of various specific ingredients such as cellulose fibre, synthetic fibre, solid lubricant, and friction modifiers [5]. In general, the composition of a paper-based friction material is schematically shown in Figure 2.

![Figure 2: General Composition of the Paper-Based Friction Material (Reproduced from [5])](image)

The main function inherent in wet clutches leads them to play a critical role in an AT. It is unavoidable that the wet clutches degrade while the AT is in operating condition. Since wet clutches are critical components, a proper condition monitoring tool should therefore be applied in order to avoid unpredictable failure. Many studies on the investigation oftribological behaviour of friction materials due to degradation have been intensively performed. In general, the studies show that the friction coefficient progressively decreases during the service-life [3,6,7], as can be seen in Figure 3. The decreasing trend remarkably enables us to forecast when a wet clutch will fail. However, in practice, the use of a friction coefficient to monitor wet clutches degradation is not easy to implement and cost expensive.

![Figure 3: Friction coefficient in function of the service-life (reproduced from [7])](image)

The degradation occurring in a wet clutch is mainly caused by both friction material and ATF degradation. The degradation mechanisms reported can occur due to mechanical degradation (adhesive wear) and thermal degradation (carbonisation) [3,8]. In reality, anyhow, both degradation mechanisms always occur simultaneously, resulting in a well known phenomenon which is called glazing. This degradation is reported to deteriorate the friction characteristics of wet clutches [6,7]. Several previous researchers experimentally revealed that the slope of the Stribeck curve (\(\mu-\nu\) curve), hereafter the slope of this curve called the Stribeck slope, in mixed-lubricating regime becomes more negative for degraded friction material than that for new friction material [9]. In contrast to the latter result, Li et al. [3] revealed that the degradation of friction material has less impact on changing the Stribeck slope. Moreover, they also found that ATF degradation has a far greater impact than friction material degradation on changing the Stribeck slope to become more negative. When the Stribeck slope becomes more negative, it is known as the loss of anti-shudder.

As a result of the glazing phenomenon, the topography of the contact surfaces changes. Previous studies [3,10] report that the surface roughness of a glazed paper-based friction material is quantitatively lower than that of a new material. Moreover, Gao and Barber [10] also report that the surface topography of a paper-based friction material has a negative skewness, and becomes more negative due to the glazing phenomenon. An extension of the Greenwood-Williamson (GW) theory developed by McCool [11] was used by the latter researchers to predict the real contact area, in order to study the contact characteristics of a wet clutch during the engagement phase. The increasing real contact area during the service-life of a wet-clutch was also analytically calculated and experimentally validated by Kimura and Otani [12], as can be seen in Figure 4.

![Figure 4: Increasing real contact area in function of service-life (reproduced from [12])](image)

Furthermore, it is also reported that due to the glazing phenomenon, the shear strength of paper-based friction materials deteriorates. This deterioration is believed to be caused by the degradation of the cellulose fibre and by pull-out of the cellulose fibre due to cyclic compression and shear stress [13]. As a consequence, the mechanical and physical properties of the friction material change [14].
Several techniques have been proposed to characterise the degradation of paper-based friction materials, such as Pressure Differential Scanning Calorimetry (PDSC), and Attenuated Total Internal Reflectance Absorbance Infrared Spectroscopy (ATR – IR) [15]. However, these techniques do not allow us to implement while the AT is still in operating condition. In other words, the online monitoring can not be implemented by using these existing techniques. The authors believe that, the two aforementioned major factors caused by glazing phenomenon might lead to a change of the contact stiffness in the so-called presliding regime which corresponds to the post-lockup phase. Obviously, if this hypothesis holds, it implies that the dynamic behaviour of the AT also changes. This change in the dynamic behaviour can be used as a means to monitor wet clutches degradation.

So far, investigations on contact stiffness characteristics of a wet clutch in the post-lockup phase were not reported yet. Therefore, this study is more focused on investigating the effects of different degradation levels of friction material on contact stiffness characteristics. Relevant models to predict the contact stiffness are also discussed in this paper. Since the surface topographies of the friction materials were not measured in this study, some relevant values given in the literatures were therefore used for the model to qualitatively predict the characteristics of the contact stiffness at different degradation levels. Furthermore, some experiments were performed in order to experimentally validate the used models. The experimental results show a qualitative agreement with the model.

2. Contact Model of Rough Surfaces

2.1 Review of Elastic Contact

In practice, all engineering surfaces are rough. This paradigm has been long realized on microscopic scale [16]. Thus, contact only occurs at asperity summits with extremely small contact area compared to the nominal area as illustrated in Figure 6. Deformation occurring at contacting region can be elastic, plastic, or elasctic-plastic depending on the nominal pressure, surface roughness, and material properties. Since the aim of this study is to qualitatively predict the contact stiffness characteristics at different degradation levels (not to accurately predict the contact stiffness), therefore, the contact occurring here is assumed to be elastic.

![Figure 6: An illustration of contact mechanism](image)

Before we further discuss the elastic contact characteristics of rough surfaces, it is important to revisit the Hertzian contact theory. According to this theory, the contact radius $a_i$, area $A_i$ and load $W_i$ are usually expressed in function of the interference $w$ as

\[ a_i = \beta \sigma^{3/2} w^{3/2}, \]  
\[ A_i = \pi \beta \sigma w, \]  
\[ W_i = \frac{4}{3} E \beta \sigma^{3/2} w^{3/2} \]

where $E$ is the combined Young’s modulus defined as

\[ E = \left( \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2} \right)^{-1} \]

$E_1$, $E_2$, and $v_1$ and $v_2$ are respectively the Young’s moduli and the Poisson’s ratios of contacting rough surfaces 1 and 2.

A Greenwood-Williamson (GW) theory [16] has been widely used to model the contact characteristics of rough surfaces. In this theory, a rough surface is assumed as a stochastic process where the surface is regarded to have asperities with simple geometrical shapes and a probability distribution function. All asperities are commonly treated as identical hemispheres having uniform radii $\beta$. This theory originally assumes that the asperities height has a Gaussian distribution $f(y)$ with zero mean given by

\[ f(y) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{y^2}{2\sigma^2} \right] \]

where $\sigma$ is the standard deviation which is equivalent to the surface roughness, and $y$ is the height of an asperity relative to the mean.
Since the summation result of two Gaussian distributions are also a Gaussian distribution. Thus, by this property, it is reasonable to consider the contact of two rough surfaces as the contact between a smooth plane and an equivalent rough surface as depicted in Figure 6. From this figure, it can be seen that, the equivalent rough surface has a reference plane (Gaussian zero plane) which is the same as the mean plane of the equivalent rough surface. At this reference plane, the height asperity is set to be zero.

If two rough surfaces are pressed together by a normal load \( W \) such that both reference planes (smooth and Gaussian zero plane) are separated by a distance \( d \) (see Figure 6), then a contact occurs at any asperity summit whose relative height to the Gaussian zero plane greater than \( d \). In accordance with the GW theory, a probability of making contact at any contacting asperity summit with height \( y \) is defined as

\[
\text{prob}(y > d) = \int_y^\infty f(y)dy
\]  

(6)

Let there are \( N \) asperity summits in total, the expected number of asperity summits in contact is

\[
n = N \int_y^\infty f(y)dy
\]  

(7)

The real contact area is given by

\[
A_r = \pi \beta N \int_y^\infty (y - d) f(y)dy
\]  

(8)

and the total elastic normal load is

\[
W = \frac{4}{3}NE\beta^2 \int_y^\infty (y - d)^{\frac{3}{2}} f(y)dy
\]  

(9)

In the GW theory, the number of asperity summits is defined as

\[
N = \eta A_n
\]  

(10)

where \( \eta \) is asperity density, \( A_n \) is the nominal contact area.

The distribution functions in Eqs. (5) – (9), for the convenience, are usually written in function of non-dimensional asperity height \( s \), where the asperity height \( y \) is normalized by the surface roughness \( \sigma (s = y/\sigma) \). Hence, the real contact area and the total normal load can be rewritten in function of normalized asperity height distribution as

\[
A_r = \pi \eta A_n \beta \sigma F_\sigma(h)
\]  

(11)

\[
W = \frac{4}{3} \pi \eta A_n E \beta^2 \sigma^{\frac{3}{2}} F_\sigma(h)
\]  

(12)

where

\[
F_\sigma(h) = \int_0^h (s - h)^{\gamma} \phi(s)ds
\]  

(13)

\( \phi(s) \) is the normalized asperity height distribution, and \( h \) is the non-dimensional separation \( h = d/\sigma \).

### 2.2 Contact of Rough Surfaces with Weibull Asperities Height Distributions

It is well known in statistics that the skewness of a Gaussian distribution is zero. As was mentioned in the previous section, it is reported that the surface topography of a paper-based friction material has a negative skewness, and therefore the latter distribution is not suitable to apply. Moreover, the reports show that a Weibull distribution is more suitable than a Gaussian distribution to characterise the surface topography of a paper-based friction material. In the present paper, the Weibull distribution is therefore used to qualitatively model the contact characteristics.

Although the summation of two Weibull distributions is not a Weibull distribution, McCool [11] analytically showed that it is still possible to extract an equivalent Weibull distribution from two different Weibull distributions.

The equivalent Weibull distribution has the same mean and variance as the sum of two latter distributions. Thus, by following the GW theory, the contact of two rough surfaces with Weibull asperities height distribution can be also considered as the contact between a smooth surface and an equivalent rough surface (GW – McCool theory). The mean value of a Weibull distribution is always positive, as will be shown later on. Therefore, as is depicted in Figure 6, the Weibull zero plane is definitely below of the Gaussian zero plane where the relative distance is \( \bar{\tau} \). Thus, the distance between the smooth and the Weibull zero planes is \( d + \bar{\tau} \).

In statistics, a Weibull distribution \( f_\omega(z) \) can be mathematically expressed as follows

\[
f_\omega(z) = \begin{cases} \frac{\kappa}{\lambda} \left(\frac{z}{\lambda}\right)^{\kappa - 1} \exp\left(-\left(\frac{z}{\lambda}\right)^\kappa\right) & z \geq 0 \\ 0 & z < 0 \end{cases}
\]  

(14)

where \( \lambda \) is a scale factor, and \( \kappa \) is a shape factor. The mean value of a Weibull distribution \( E(z) \) is given by

\[
E(z) = \bar{\tau} = \lambda B_1
\]  

(15)

Both aforementioned parameters \( \lambda \) and \( \kappa \) are positive, and respectively related to the surface roughness \( \sigma \) and the skewness \( (R_s) \). The relations can be seen in the following expressions (the detailed explanation can be found in [11])

\[
\sigma = \lambda (B_2 - B_3^2)^{\frac{1}{2}}
\]  

(16)

\[
R_s = \frac{B_1 - 3B_2B_3 + 2B_3^2}{(B_2 - B_3^2)^{\frac{3}{2}}}
\]  

(17)
which is expressed as follows

\[ B_\kappa = \Gamma \left( 1 + \frac{n}{\kappa} \right) \]  

(18)

For a Weibull distribution, one can show that the normalized asperity height distribution is given by [10]

\[ \varphi_n(s_n) = \left( \frac{\kappa}{\lambda} \right)^n \exp \left[ - \left( \frac{s_n}{\lambda} \right)^\kappa \right] \]  \( z > 0 \)  

(19)

\( s_W \) is the non-dimensional asperity height relative to the Weibull zero plane, where the relative height \( z \) is normalized by the surface roughness \( \sigma (s_W = z/\sigma) \). Furthermore, in accordance with the GW – McCool theory, hence, Eqs. (11) – (13) can be rewritten as

\[ A_n = \pi \eta \beta \sigma f_{h_n, \kappa} \]  

(20)

\[ W = \frac{4}{3} \pi \eta \beta \sigma v^{1/2} f_{h_n, \kappa} \]  

(21)

\[ F_{n, \kappa}(\hat{h}_n, \kappa) = \int_{\hat{h}_n - \hat{h}_n} \varphi_n(s_n)ds_n \]  

(22)

where \( \hat{h}_n \) is the non-dimensional distance between the smooth and the Weibull zero planes given by

\[ \hat{h}_n = \frac{d + \sigma}{\sigma} = \frac{\hat{h}}{\sigma} = \frac{B_1}{(B_2 - B_1)^{1/2}} \]  

(23)

2.3 Real Contact Area

Gao et al. [10,17] firstly implemented the Weibull distribution to predict the real contact area of a paper-based wet clutch during engagement phase. Based on Eq.(20), they derived a mathematical expression of the real contact area which can be expressed as follows

\[ A_n = \alpha^{-1} \pi \eta \beta \sigma \left[ \frac{A_n}{\sigma} \Gamma \left( 1 + \frac{1}{\kappa} \right) \left( 1 + \frac{1}{\kappa} \right) \left( \frac{\hat{h} \sigma}{A_n} \right)^{1\kappa} \right] \]  

(24)

\[ \hat{h} = \exp \left[ - \left( \frac{\hat{h} \sigma}{\lambda} \right)^{1\kappa} \right] \]  

where \( \alpha \) is a correction factor which is given in [10], and \( P(.,.) \) is the incomplete gamma function.

Meanwhile, the real contact area predicted by using a Gaussian distribution can be expressed as follows [18]

\[ A_n = \pi \eta \beta \sigma \left[ \frac{1}{2\pi} \exp \left( - \frac{\hat{h}^2}{2} \right) + \frac{1}{2} \exp \left( \frac{\hat{h}}{\sqrt{2}} \right) - 1 \right] \]  

(25)

2.4 Normal Contact Stiffness

After engagement, the film thickness of the ATF reaches the minimum value, and therefore, the thickness of this film is reasonably negligible compared to the separation of two reference planes. Hence, it can be assumed that the contact behaviour as close to the unlubricated contact condition (Hertzian contact).

For the unlubricated elastic contact, the normal contact stiffness of two asperities can be calculated by differentiating Eq. (3) with respect to the interference \( w \)

\[ k_n = \frac{dW_n}{dw} = 2E\beta^2 \sigma^{1/2} \]  

(26)

By extending Eq. (26) with the GW – McCool theory, the total normal contact stiffness of two contacting rough surfaces \( K_n \) can be thus expressed as follows

\[ K_n = 2\pi \eta \beta \sigma v^{1/2} F_{n, \kappa}(\hat{h}_n, \kappa) \]  

(27)

From Eq. (21) and (27), the normal contact stiffness can be also rewritten in function of the normal load as [19,20]

\[ K_n = \frac{3W}{2\sigma} F_{n, \kappa}(\hat{h}_n, \kappa) \]  

(28)

where \( F_{n, \kappa}(\hat{h}_n, \kappa) \) is given by

\[ F_{n, \kappa}(\hat{h}_n, \kappa) = \frac{F_{\infty}(\hat{h}_n, \kappa)}{F_{\infty}(\hat{h}_n, \kappa)} \]  

(29)

From Eq. (28), it can be seen that the normal contact stiffness is linearly proportional to the applied normal load. In addition, the approximation of Eq. (29) is given in the Appendix A.

2.5 Tangential Contact Stiffness

If two contacting hemispherical asperities are subjected to a tangential force \( Q_t \), then micro-slip \( \delta \) will be observed (see Figure 7). The relationship between the micro-slip and the tangential force was independently studied by Cattaneo and Mindlin [21] as follows

\[ Q_t = \mu W - \frac{4}{3} \mu E\beta \gamma^{1/2} \]  

(30)

where \( \mu \) is a constant friction coefficient, and \( \gamma \) is the combined elastic material constant defined as [19]

\[ \gamma = 4G/\mu E \]  

(31)

with \( G \) is the combined shear modulus defined as

\[ G = \left( \frac{2 - \nu_1}{G_1} + \frac{2 - \nu_2}{G_2} \right)^{-1} \]  

(32)

\( G_1 \) and \( G_2 \) are the shear moduli of contacting rough surfaces 1 and 2.
Let the two contacting hemispherical asperities lie in an infinitesimal annulus with radius $r$, and thickness $\Delta r$, as depicted in Figure 7.

![Figure 7: A schematic of contacting friction disc with an infinitesimal annulus. The annulus is considered as a nominally flat surface with area equal to $2\pi r \Delta r$](image)

Since $\delta_0 = r \delta_0$, thus, the relative torsion $T_\gamma$ with respect to the annulus centre $C$, which is required to generate micro-slip is given by

$$T_\gamma = \tau Q_\gamma = \frac{4}{3} \mu E \beta^{1/3} r \left(1 - \gamma \theta \tau \right)^{1/2}$$  \hspace{1cm} (33)

By differentiating Eq. (33) with respect to angular displacement $\theta$, one can show that the tangential contact stiffness of the annulus can be expressed as follows

$$k_i = \frac{dT_\gamma}{d\theta} = 2 \mu E \beta^{1/3} r \left(1 - \gamma \theta \tau \right)^{1/2}$$

$$= 8G \beta^{1/3} r \left(1 - \gamma \theta \tau \right)^{1/2}$$  \hspace{1cm} (34)

Furthermore, by extending Eq. (34) with the GW – McCool theory, the total tangential contact stiffness of the infinitesimal annulus $\Delta K_i$, can be thus expressed as (the derivation is given in the Appendix B)

$$\Delta K_i = 16 \pi G \beta^{1/3} \sigma^{1/2} F_{\gamma}^{\infty} \left\{ \hat{h}_w + \gamma \theta \vec{\sigma} \right\} \dot{\tau} \Delta r$$

$$= 16 \pi G \beta^{1/3} \sigma^{1/2} F_{\gamma}^{\infty} \left\{ \hat{h}_w + \gamma \theta \vec{\sigma} \right\} \dot{\tau} r \Delta r$$  \hspace{1cm} (35)

where $\dot{\tau} = r / \sigma$ is a non-dimensional radius.

Finally, the total tangential contact stiffness $K_i$ of the friction disc with inner radius $R_i$ and outer radius $R_o$ can be computed by integrating the annulus from $R_i$ to $R_o$

$$K_i = 16 \pi G \beta^{1/3} \sigma^{1/2} F_{\gamma}^{\infty} \left\{ \hat{h}_w + \gamma \theta \vec{\sigma} \right\} r \Delta r$$  \hspace{1cm} (36)

Note that, according to Eq. (36), the tangential contact stiffness is not only dependent on the normal displacement, but also on the tangential displacement.

For a relatively thin disc, the term $F_{\gamma}^{\infty} \left\{ \hat{h}_w + \gamma \theta \vec{\sigma} \right\}$ in Eq. (36) can be considered to be independent of the non-dimensional radius. Hence, the latter equation can be approximated as follows

$$K_i = 8 \pi G \beta^{1/3} \sigma^{1/2} R_o \Delta r F_{\gamma}^{\infty} \left\{ \hat{h}_w + \gamma \theta \vec{\sigma} \right\}$$  \hspace{1cm} (37)

where $A_\delta = (R_i + R_o) / 2$, $\Delta R = R_o - R_i$, and $\hat{R}_w = R_w / \sigma$ are the nominal area, the mean radius, the thickness, and the non-dimensional mean radius of the thin disc, respectively.

From Eq. (21) and (37), the tangential contact stiffness can be rewritten in function of the normal load as follows

$$K_i = \frac{6 G \beta^{1/3} \sigma^{1/2} \pi R_i R_o}{E \sigma} F_{\gamma}^{\infty} \left\{ \hat{h}_w + \gamma \theta \vec{\sigma} \right\}$$

$$= \frac{6 G \beta^{1/3} \sigma^{1/2} \pi R_i R_o}{E \sigma} \left\{ \hat{h}_w + \gamma \theta \vec{\sigma} \right\}$$  \hspace{1cm} (38)

### 3. Analytical Simulation of Degradation Effects

In this section, analytical simulations based on the models previously presented were carried out. The simulations are mainly intended to numerically investigate the effect of the friction material degradation levels on the contact stiffness characteristics in the post-lockup phase. There are several assumptions used in the simulations as follows: the contact occurs between a friction disc and a separator disc, the ATF effect in the contact is neglected, the geometry of the friction disc is thin without grooves, and the separator disc has a smooth surface.

The calculations of the normal and the tangential contact stiffnesses are respectively based on Eqs. (28) and (38). In the calculations, the geometrical parameters used are based on the dimension of the specimens used in the experiment. This experiment is referred to SAE #2 tests and will be briefly discussed in the experimental section. Moreover, several material parameters given in reference [10] are used, and the others are obtained from measurement. The parameters used in the calculation are listed in Table 1.

| Inner diameter of friction material, $d_i$ (m) | 0.1115 |
| Outer diameter of friction material, $d_o$ (m) | 0.1173 |
| Asperity radius of friction material, $\beta$ (um) | 500 |
| Asperity density of friction material, $\eta$ (m$^{-2}$) | 3x10$^3$ |
| Young’s modulus of friction material, $E_i$ (MPa) | 45 |
| Poisson’s ratio of friction material, $\nu_i$ | 0.2 |
| Shear modulus of friction material, $G_i$ (MPa) | 18.75 |
| Young’s modulus of separator, $E_s$ (MPa) | 203 |
| Poisson’s ratio of separator, $\nu_s$ | 0.3 |
| Shear modulus of separator, $G_s$ (MPa) | 78 |
| Friction coefficient, $\mu$ | 0.15 |

Three different degradation levels were considered in the simulation, namely new, run-in, and glazed materials. The topographical parameters given in [10] corresponding to the degradation levels of friction material were used, as tabulated in Table 2. In the table, it can be seen that, the surface roughness of the friction material decreases and the skewness becomes more negative as the degradation progresses further.
As was previously discussed, the friction coefficient and the shear strength of the friction material deteriorate due to the degradation (see Figure 3 and Figure 5). In the material mechanics, it is well known that the shear strength $S_t$ is proportional to the shear modulus. Thus, it can be concluded that due to the degradation, the shear modulus of the friction material $G_t$ deteriorates. As a result, the combined shear modulus $G$ also deteriorates. However, according to Eq. (28), both deteriorations do not affect the normal contact stiffness; instead, they affect the tangential contact stiffness in accordance with Eq. (38).

In the tangential contact stiffness simulations, the friction coefficient of the new material used is based on the value given in Table 1. It is assumed that the friction coefficient of the run-in material is 0.90 times the one of the new material, and the friction coefficient of the glazed material is 0.85 times the one of the new material. To simulate the effect of the shear modulus on the tangential contact stiffness, two cases were considered. The consideration of both cases was based on a possibility that during run-in, the combined shear modulus may change. In the simulations, the combined shear modulus of the new material used is based on the values given in Table 1. In the first case, it is assumed that the combined shear modulus of the run-in material remains the same as the one of the new material. In the second case, it is assumed that the combined shear modulus of the run-in material decreases to 0.8 times the one of the new material. For both cases, the combined shear modulus of the glazed material is assumed to be the same, which is 0.5 times the one of the new material. The summary of the friction coefficient and the combined shear modulus used in the simulations are given in Table 3.

Table 3: The combination of the friction coefficient and the combined shear modulus used in the simulations for the new, run-in, and glazed materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Friction coefficient $\mu$</th>
<th>Combined shear modulus $G$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Case # 1</td>
</tr>
<tr>
<td>New</td>
<td>$1.00 \times \mu_{\text{new}}$</td>
<td>$1.0 \times G_{\text{new}}$</td>
</tr>
<tr>
<td>Run-in</td>
<td>$0.90 \times \mu_{\text{new}}$</td>
<td>$1.0 \times G_{\text{new}}$</td>
</tr>
<tr>
<td>Glazed</td>
<td>$0.85 \times \mu_{\text{new}}$</td>
<td>$0.5 \times G_{\text{new}}$</td>
</tr>
</tbody>
</table>

Since the contact stiffness depends on the static normal load, it is therefore necessary to study the contact stiffness characteristics at different static normal loads. These normal loads were chosen in such a way that the contact pressure on the friction disc in the simulation should represent the actual pressure applied on the friction discs used in the SAE#2 tests. In the wet clutch used for the aforementioned tests, the apparent contact area of the hydraulic cylinder is larger than the apparent contact area of the friction disc. Hence, the actual contact pressure applied on the friction disc is larger than the pressure applied on the hydraulic cylinder.

Figure 8 shows a representative pressure applied on the wet clutch and the relative speed between the input and output shafts during a duty cycle obtained from the SAE#2 test. It can be seen in the figure that, at the lockup where the relative speed reaches zero, the pressure applied on the cylinder is ~6 bar. Equivalently, the actual contact pressure applied on the friction disc at the lockup is ~7.2 bar. In accordance with the specified geometry of the friction disc given in Table 1, thus, the static normal load at the lockup is ~726 N. Nevertheless, in the simulations, 5 different static normal loads were used. The range of the loads was chosen such that they lie in the range of the pre- and post-lockup in the SAE#2 test and should not exceed the maximum allowable normal force in the contact stiffness identification. These loads are 200, 400, 600, 800, 1000 N and equivalent to an actual pressure on the friction disc of 2, 4, 6, 8, and 10 bar respectively.

3.1 Simulation Results of the Normal Contact Stiffness

Prior to the calculation of the normal contact stiffness at a given normal load $W$ and degradation level, the non-dimensional distance $\hat{h}_W$ has to be firstly determined. The latter distance is determined by using Eq. (21). Afterwards, the normal contact stiffness can be computed by using Eq. (28). At a certain normal load, the value of $\hat{h}_W$ changes in function of the degradation level, as can be seen in Figure A1 of Appendix A.

Figure 9 shows the simulation results of the normal contact stiffness at different degradation levels and different static normal loads. The figure shows that, the normal contact stiffness increases with the increase of the static normal load as predicted by Eq. (28).
Moreover, one can see in the figure that, in the given range of the static normal loads, the normal contact stiffness definitely increases as the friction material degradation progresses. The discrepancy between the normal contact stiffness of the run-in material and the one of the new material is relatively large. However, as the friction material degradation becomes more severe, the discrepancy between the normal contact stiffness of the glazed material and the one of the run-in material is not quite large.

Figure 9: The simulation of degradation effects on the normal contact stiffness

### 3.2 Simulation results of the tangential contact stiffness

Slightly different to the latter calculations, the non-dimensional distance $h_W$ and the tangential displacement $\theta$ have to be firstly determined before calculating the tangential contact stiffness. The value of the tangential displacement must be chosen in such a way that the micro-slip is observed. In the simulations, $\theta = 1 \times 10^{-5}$ rad was used.

The simulation results of the tangential contact stiffness at different degradation levels and different static normal loads are given in Figure 10, and Figure 11. As is predicted by Eq. (38), the tangential contact stiffness depicted in both figures increases with the increase of the static normal load. In addition, both figures show different characteristics, especially at relatively high normal load.

From Figure 10 it can be concluded that, at relatively low normal loads, the tangential contact stiffness of the run-in material is lower than that of the new material. On the other hand, at relatively high normal loads, the tangential contact stiffness of the run-in material is higher than that of the new material.

Figure 10: The simulation of degradation effects on the tangential contact stiffness, case # 1 ($G_{\text{run-in}} = G_{\text{new}}$, $G_{\text{glazed}} = 0.5G_{\text{new}}$, $\mu_{\text{run-in}} = 0.9\mu_{\text{new}}$, $\mu_{\text{glazed}} = 0.85\mu_{\text{new}}$)

In contrast to the latter result, Figure 11 shows that the tangential contact stiffness of the run-in material is lower than that of the new material in all given loads range. However, both latter figures show that the tangential contact stiffness of the glazed material is definitely lower than that of the new material.

Figure 11: The simulation of degradation effects on the tangential contact stiffness, case # 2 ($G_{\text{run-in}} = 0.8G_{\text{new}}$, $G_{\text{glazed}} = 0.5G_{\text{new}}$, $\mu_{\text{run-in}} = 0.9\mu_{\text{new}}$, $\mu_{\text{glazed}} = 0.85\mu_{\text{new}}$)

### 4. Experiment

Two different tests were performed in this study, namely a durability test, and contact stiffness identification. The durability test was performed on a SAE#2 test setup in order to accelerate the service-life of the friction materials, and the contact stiffness identification was performed on a newly developed tribometer.
4.1 Experimental Setups

4.1.1 Description of the SAE#2 Test

The description of the SAE#2 test setup is briefly described in this paper. Basically, this test setup consists of input and output electric motors (1 & 9), input and output flywheels (3 & 7), and a wet clutch (4) as schematically depicted in Figure 12. Both flywheels are independently driven by the electric motors. Moreover, both motors are driven at the same speed, but in opposite direction. The input and output flywheels velocities are measured by optical encoders (2 & 8). At a desired relative speed (~ 4000 rpm), both motors are powered-off and the wet clutch is immediately closed by applying a pressurised oil controlled by a valve (5) onto the clutch. As the oil pressure increases, the relative speed decreases as can be seen in Figure 8.

4.1.2 Description of the Tribometer

The tribometer used in this study as shown in Figure 13, consists of the following main components: (1) shaker (dynamic load), (2) shaker holder, (3) frame, (4) vertical guide ways, (5) cantilever, (6) static load with cantilever, (7) axial bearing, (8) Direct Drive motor, (9) ball spline bearing, (10) shaft, (11) bellow coupling, (12) tub containing the two friction discs (and oil), (13) capacitive displacement sensors (3 off), (14) three ring dynamometers for force and torque measurement, (15) base plate.

The main objective of the tribometer is to characterise the friction, in dry and lubricated conditions, occurring between friction and separator discs from an AT [22]. However, by a small modification, the functionality of the tribometer can be extended for identifying the contact stiffness characteristics. The static normal load was applied by a pneumatic actuator added on top of the shaker. This additional configuration allows us to easily identify the contact stiffness at different static normal loads and angular positions.

To identify the contact stiffness, the friction torque, normal force, and the relative displacements in both normal and tangential directions should be measured as accurately as possible. This is briefly discussed as follows.

The friction torque and normal force are measured by a 6 DOF measuring table with 3 ring dynamometers comprising strain gauges. The maximum normal force and torque which can be applied to the measuring table are respectively 1293 N and 15 Nm.

To generate a micro-slip in the tangential direction, a Direct Drive motor from Dynaserv YOKOGAWA Precision is used. The motor has a built-in encoder which is used for the angular position measurement. The resolution of the encoder is 163840 pulses/revolution. The motor is actuated by a proportional-derivative (PD) position controller.

To generate a dynamic normal force, a Philips PR 9270 shaker is used. The shaker can generate a force of $35.7 \times I_{\text{eff}}$ [N], where $I_{\text{eff}}$ is the effective current in Ampere. The shaker is driven in open loop and has a frequency bandwidth of $0 – 10000$ Hz.

The resulting normal displacement is measured by the 3 capacitive displacement sensors. This is done by averaging the signals coming out of the capacitive sensors.

A CLP1103 dSPACE system is used for the data acquisition. The friction torque measurement is connected to the first three ADC channels with a resolution of 16 bit. The other channels have a resolution of 12 bit. The other ADC inputs contain three channels for the normal force, three channels for the normal displacement, one encoder connection and six digital I/O to reset the torque and the normal force (auto-zero). Two DAC channels are used for the control inputs, one for the motor and one for the shaker.

4.2 Procedures of the Contact Stiffness Identification

The applied normal loads used in the identification of the contact stiffness (normal and tangential) are chosen the same as the static normal loads used in the simulation. Moreover, the contact stiffness was also identified at 8 different angular positions.

In the identification of the tangential contact stiffness, at a given static normal and angular position, the motor is
actuated with a dynamic command signal. In order to avoid undesired effects, the tangential displacement must be chosen in such a way that micro-slip is observed, and also the excitation frequency should not affect the dynamic behaviours of the tribometer. In this study, the command signal applied to the motor is sinusoidal with experimentally predefined amplitude of $4 \times 10^{-3}$ rad, and excitation frequency of 0.5 Hz.

In the normal contact stiffness identification, at a given static normal load and angular position, the shaker is actuated with a dynamic command signal. The current signal applied to the shaker is also sinusoidal with amplitude of 0.25 Ampere with frequency of 0.5 Hz. This current amplitude is approximately ~9 N.

### 4.3 Specimens

Three identical paper-based specimens were used in the study. These specimens have 2 sets of 9 parallel grooves which are perpendicular to each other, as can be seen in Figure 14. One specimen is totally new, and the others are degraded. Since, the degraded ones have been exposed to respectively 10,100 and 20,000 duty cycles, they can be considered as glazed materials. These two degraded materials, namely Glazed 1 and Glazed 2, are obtained from the durability tests on the SAE#2 test setup. While, the run-in material was obtained by running the new specimen on the tribometer for ~5 hours with applied normal load of ~1200 N and relatively constant sliding speed of ~2.5 rad/s which is equivalent to 500 actual duty cycles in the SAE#2 tests.

Prior to the contact stiffness identification on the tribometer, some preparations for the specimens are required. The friction lining in one side of the specimens must be completely removed, and then the specimens are machined such that the inner diameter is not less than 0.11 m, in order to be able to mount them on the tribometer. Afterwards, the surface without friction lining of each specimen is glued on a dedicated disc holder which is specially designed for the tribometer. The nominal (apparent) contact area of each specimen after preparation, as depicted in Figure 15, is approximately $1 \times 10^{-3}$ m².

### 5. Results and Discussion

Some typical results obtained from the experiments are given in Figure 16, and Figure 17, and show a relatively linear behaviour with a small hysteresis loop, especially for the tangential contact stiffness. A simple linear regression method is therefore used here to identify the contact stiffnesses by calculating the mean slope of the hysteresis curves.
The resulting normal and tangential contact stiffness for different normal loads and different degradation levels are given in Figure 18 and Figure 19. As was previously expected from the simulations, in general, the normal and tangential contact stiffnesses increase with an increasing static normal load. This can be explained by the fact that, the separation between two contacting surfaces decreases with an increasing normal load. Consequently, the real contact area and the number of asperities in contact also increase.

As was investigated in [10,17], due to high negative skewness, the real contact area of the glazed materials is higher than that of the new and the run-in materials. As a result, at a given static normal load, the glazed materials have more contact zones compared to the new and run-in materials. Therefore, the normal contact stiffness of the glazed material is higher than that of the others, as has been seen in Figure 18. This figure shows that, the normal contact stiffness increase, with the increase of degradation levels. Moreover, this figure also shows that the glazed friction materials have higher normal contact stiffness than the new and run-in materials.

Figure 18: The experimental normal contact stiffness at different degradation levels

In contrast to the latter results, the tangential contact stiffness shows opposite characteristics; the tangential contact stiffness decreases with increasing the degradation level which confirms the simulation results.

However, at low static normal load (~200 N) the tangential contact stiffness of the new and run-in material is lower than that of the glazed material. It is obvious that, at low static normal load, the separation between the two contacting surfaces is relatively large. However, due to the high negative skewness and low roughness of the glazed material, the separation of the glazed material is much smaller than that of the run-in and the new material. Therefore, at the low static normal load, the film fluid formed in between of the two contacting surfaces is not really dominant for the glazed material, which is in contrast to the run-in and the new material where the film fluid formed plays a dominant role.

Figure 19: The experimental tangential contact stiffness at different degradation levels

6. Conclusions
Stochastic models of the normal and tangential contact stiffness using the GW – McCool theory have been presented in this paper. The models show a qualitative agreement with the experimental results. As the material degradation level progress, both tangential and normal contact stiffnesses deviate from the initial values (at new material). The normal contact stiffness shows an increasing trend; in contrast, the tangential contact stiffness shows a decreasing trend. Remarkably, the aforementioned trends enable us to develop a condition monitoring strategy on wet clutches. This can be done by a means to monitor the contact stiffness based on dynamic behaviours.

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Appendix A

From Eq. (19), the normalized Weibull distribution can be rewritten as

$$ \phi_w(s_w) = \frac{\alpha_s}{\rho^s} \exp \left[ -\left( \frac{s_w}{\rho} \right)^s \right] \tag{A1} $$

where $\rho = \frac{\sigma}{h}$

Since the contact occurs if and only if $s_w > h_W$, or for the convenience it may also be written as $h_W/s_w < 1$, then $F_{W_w}(h_W, \kappa)$ can be computed as follows

$$ F_{W_w}(h_W, \kappa) = \int_{s_w}^{h_W} \left( 1 - \frac{h}{s} \right)^{\frac{1}{2}} \phi_w(s_w) ds_w \tag{A2} $$

as $h_W/s_w < 1$, the term in the brackets of Eq. (A2) can be expanded using an infinite Taylor series as follows [23]

$$ \left( 1 - \frac{h}{s} \right)^{\frac{1}{2}} = 1 - \sum_{n=1}^{\infty} \frac{(2n)!}{(2n-1)!} \left( \frac{h}{s} \right)^n \tag{A3} $$

Substituting Eq. (A3) into Eq. (A2) results in

$$ F_{W_w}(h_W, \kappa) = \int_{s_w}^{h_W} \left( 1 - \sum_{n=1}^{\infty} \frac{(2n)!}{(2n-1)!} \left( \frac{h}{s} \right)^n \right) \phi_w(s_w) ds_w $$

$$ = \int_{s_w}^{h_W} s_w^{-\frac{1}{2}} \phi_w(s_w) ds_w - \sum_{n=1}^{\infty} \frac{(2n)!}{(2n-1)!} \int_{s_w}^{h_W} s_w^{-\frac{1}{2}} \phi_w(s_w) ds_w \tag{A4} $$

let

$$ u = \left( \frac{s_w}{\rho} \right)^s \Rightarrow s_w = \rho \cdot u^{-\frac{1}{s}} \tag{A5} $$

thus

$$ du = \frac{\alpha_s}{\rho^s} ds_w \tag{A6} $$

Substituting Eqs. (A1), (A5), and (A6) into Eq. (A4) results in

$$ F_{W_w}(h_W, \kappa) = \rho^{\frac{1}{s}} \int_{0}^{u_{\rho}} \exp(-u) du $$

$$ - \sum_{n=1}^{\infty} \frac{(2n)!}{(2n-1)!} \int_{0}^{u_{\rho}} \exp(-u)^n du $$

$$ = \rho^{\frac{1}{s}} \int_{0}^{u_{\rho}} \exp(-u) du - \sum_{n=1}^{\infty} \frac{(2n)!}{(2n-1)!} \int_{0}^{u_{\rho}} \exp(-u)^n du \tag{A7} $$

For the convenience, Eq. (A7) can be rewritten as

$$ F_{W_w}(h_W, \kappa) = \rho^{\frac{1}{s}} F_{W_w} - \sum_{n=1}^{\infty} \frac{(2n)!}{(2n-1)!} \rho^{\frac{n}{s}} F_{W_w}^{n} \tag{A8} $$

where

$$ F_{W_w} = \int_{0}^{u_{\rho}} \left( 1 + \frac{1}{2s} \right) - \left( 1 + \frac{1}{2s} \right) \left( \frac{h}{\rho} \right)^s \tag{A9} $$

and

$$ F_{W_w}^{n} = \int_{0}^{u_{\rho}} \left( 1 + \frac{1-2n}{2s} \right) - \left( 1 + \frac{1-2n}{2s} \right) \left( \frac{h}{\rho} \right)^{s n} \tag{A10} $$

In similar way, $F_{W_w}(h_w, \kappa)$ can be computed as follows

$$ F_{W_w}(h_w, \kappa) = \int_{s_w}^{h_w} \left( s_w^{-\frac{1}{2}} \phi_w(s_w) ds_w \right) $$

$$ = \int_{s_w}^{h_w} s_w^{-\frac{1}{2}} \phi_w(s_w) ds_w \tag{A11} $$

Again, by using Taylor series expansion and by the help of Eq. (A3), the term in the brackets of Eq. (A11) can be written as

$$ \left( 1 - \frac{h}{s} \right)^{\frac{1}{2}} = 1 - \sum_{n=1}^{\infty} \frac{(2n)!}{(2n-1)!} \left( \frac{h}{s} \right)^n $$

$$ = 1 - \frac{h}{s} - \sum_{n=1}^{\infty} \frac{(2n)!}{(2n-1)!} \left( \frac{h}{s} \right)^n \tag{A12} $$

One can show that, the Eq. (A12) can be simplified as

$$ \left( 1 - \frac{h}{s} \right)^{\frac{1}{2}} = 1 - \frac{3}{2} \Phi \left( \frac{h}{s} \right)^s + \sum_{n=2}^{\infty} \Phi_n \left( \frac{h}{s} \right)^n \tag{A13} $$

where

$$ \Phi = \frac{(2n-2)}{(2n-3)(n-1)!} \frac{(2n)!}{(2n-1)! n!} \tag{A14} $$
Substituting Eq. (A13) into Eq. (A11) results in
\[ F_{\nu}^{\nu}\left(h_w, \kappa \right) = \int_{z_w}^{z_w + \frac{3h_w}{2}} \frac{3h_w}{2} \int_{x_w}^{x_w + \frac{3h_w}{2}} p_w x_w dx_w + \sum_{n=2}^{\infty} \Phi \left( \Phi_\nu \right) \int_{z_w}^{z_w + \frac{3h_w}{2}} \int_{x_w}^{x_w + \frac{3h_w}{2}} p_w x_w dx_w \tag{A15} \]
and again, substituting Eqs. (A1), (A5), and (A6) into Eq. (A15), one can show that
\[ F_{\nu}^{\nu}\left(h_w, \kappa \right) = \rho \int_{u}^{u + \frac{3h_w}{2}} \frac{3h_w}{2} \int_{u}^{u + \frac{3h_w}{2}} p_w u_w du + \sum_{n=2}^{\infty} \Phi \left( \Phi_\nu \right) \int_{u}^{u + \frac{3h_w}{2}} \int_{u}^{u + \frac{3h_w}{2}} p_w u_w du \tag{A16} \]
as the same procedure in Eq. (A7), hence, Eq. (A16) can be reformulated as follows
\[ F_{\nu}^{\nu}\left(h_w, \kappa \right) = \rho \int_{u}^{u + \frac{3h_w}{2}} \frac{3h_w}{2} \int_{u}^{u + \frac{3h_w}{2}} p_w u_w du + \sum_{n=2}^{\infty} \Phi \left( \Phi_\nu \right) \int_{u}^{u + \frac{3h_w}{2}} \int_{u}^{u + \frac{3h_w}{2}} p_w u_w du \tag{A17} \]
where
\[ F_{\nu}^{\nu} = \Gamma \left(1 + \frac{3}{2\kappa} \right) \left( \frac{h_w}{\rho} \right) \]  
\[ F_{\nu}^{\nu} = \Gamma \left(1 + \frac{1}{2\kappa} \right) \left( \frac{h_w}{\rho} \right) \]  
\[ F_{\nu}^{\nu} = \Gamma \left(1 + \frac{3}{2\kappa} \right) \left( \frac{h_w}{\rho} \right) \]  
\[ F_{\nu}^{\nu} = \Gamma \left(1 + \frac{1}{2\kappa} \right) \left( \frac{h_w}{\rho} \right) \]  
and
\[ F_{\nu}^{\nu} = \Gamma \left(1 + \frac{3-2n}{2\kappa} \right) \left( \frac{h_w}{\rho} \right) \]  
\[ F_{\nu}^{\nu} = \Gamma \left(1 + \frac{1}{2\kappa} \right) \left( \frac{h_w}{\rho} \right) \]  
\[ F_{\nu}^{\nu} = \Gamma \left(1 + \frac{3-2n}{2\kappa} \right) \left( \frac{h_w}{\rho} \right) \]  
\[ F_{\nu}^{\nu} = \Gamma \left(1 + \frac{1}{2\kappa} \right) \left( \frac{h_w}{\rho} \right) \]  
Finally, from Eq. (10) and (22), the latter equation can be rewritten as
\[ \Delta K_n = 16 \pi \eta G \beta \gamma^2 \int_{h_w + \gamma \delta \kappa}^{h_w + \gamma \delta \kappa + \Delta r} \left( h_w + \gamma \delta \kappa \right) r^2 dr \tag{B7} \]
where \( F_{\nu}^{\nu} \left(h_w + \gamma \delta \kappa, \kappa \right) \) is calculated by using Eq. (A8).