Pre-filtered Hankel Total Least Squares method for condition monitoring of wet friction clutches

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Abstract

Wet friction clutches play a critical role in a transmission system. The clutches progressively degrade while the transmission is under operational condition. If the clutches unexpectedly fail, then the complete transmission will fail. To avoid unexpected failures, a proper condition monitoring tool extended with a lifetime prediction method must be implemented. In order to achieve this, appropriate parameters representing the degradation level must be determined. Our previous study reveals that tangential contact stiffness of the clutch in the post-lockup phase changes in function of the clutch degradation level. Accordingly, the degradation can be monitored via changes of the dynamic behaviour of a transmission which is expressed by changes of the torsional natural frequencies and damping ratios of the transmission. Things become more complicated by the fact that, since the applied normal load in the post-lockup phase is not constant, the contact stiffness varies. As a consequence, the transmission behaves like a time-variant system. This behaviour is manifested by varying high torsional vibration natural frequencies. However, the dominant low-frequency of the post-lockup torsional vibration signal is relatively constant. In this work, the dominant undamped natural frequency and the corresponding damping ratio are proposed as relevant features. The features are extracted by using pre-filtered Hankel Total Least Squares (HTLS) method. The results show that the proposed features exhibit clear trends related to the clutch degradation.

Keywords: Hankel Total Least Squares (HTLS), wet friction clutches monitoring, contact stiffness, torsional vibration.

1. Introduction

Wet friction clutches principally work based on utilisation of friction, generated in lubricated contacting surfaces. The clutch is lubricated by an Automatic Transmission Fluid (ATF) having a function as a cooling lubricant cleaning the contacting surfaces...
and giving smoother performance and longer life. However, the presence of the ATF in the clutch reduces the friction coefficient. In some applications where high power is needed, the clutch is commonly designed with several friction and separator discs. This configuration is known as a multi-disc clutch as can be seen in Figure 1, wherein the friction discs are mounted to the input shaft by splines, and the separator discs are mounted to the output shaft by lugs.

![Figure 1 A schematic view of a multi-disc wet friction clutch](image)

Wet friction clutches play a critical role in AT systems. It is unavoidable that they progressively degrade while the AT system is being under operational condition. If they unexpectedly fail, then the whole transmission will fail. To avoid the unexpected failure, a proper condition monitoring tool extended with a lifetime prediction method is considered to be the best approach. In order to achieve this, appropriate parameters representing the fault types and degradation level must be determined. Hereafter, these parameters will be referred to as relevant features.

The friction coefficient has been long considered as a potential parameter for monitoring the clutch degradation. Without any doubt, the monitoring of the friction coefficient which is computed from both transmitted torque and applied normal force signals can be performed online. Several studies have been conducted to investigate the friction coefficient behaviour during the lifetime of the clutch \(^{(1,2,3)}\). In general, they show that the friction coefficient progressively decreases with a clear trend. The drawback of using this approach nevertheless is that at least two sensors are absolutely needed to compute the coefficient. As a consequence, the price to be paid to implement this approach is possibly expensive. In addition, in real-life applications, both sensors are probably difficult to install in the transmissions.

Alternatively, in our previous study \(^{(4)}\), it is revealed that the friction material degradation of the clutch can also be observed via the change of contact stiffness in the post-lockup phase both in normal and tangential directions. Moreover, they also show that the contact stiffness depends on the applied normal load. The change of the contact stiffness manifested in the change of the dynamic behaviour of the transmissions can therefore be considered as an indicator or a symptom that the clutch has degraded. This new alternative gives an advantage to the users in that a single sensor can be possibly used to monitor the clutch degradation either in tangential or normal direction.
In the post-lockup phase, it is observed that the typical torsional vibration is transient and relatively short \(^{(3,5)}\). Moreover, in this particular phase the normal load applied on the clutch increases, in most of cases \(^{(3,5)}\). This means that the contact stiffness is varying, thus the transmission systems obviously behave like a time-variant system. The Time-Frequency Representation (TFR) of the torsional vibration signal in our other previous study \(^{(5)}\) shows that frequencies are not constant at high vibration mode(s). However, the low torsional natural frequency is relatively constant. In the remaining of the paper, this frequency is called the dominant natural frequency. The dominant natural frequency and the corresponding damping ratio are proposed as relevant features for monitoring the clutch degradation.

Accurate estimation of the relevant features is an essential step for clutch monitoring. To accurately estimate the features in the abovementioned circumstances, the use of a time-domain identification method, namely Hankel Total Least Squares (HTLS) is proposed \(^{(5)}\). Furthermore, it is also proposed that the torsional vibration signal is first filtered by a band-pass filtering technique prior to the estimation of the parameters. The bandwidth of the filter is chosen such that the torsional mode(s) with relatively constant frequency lies in the frequency range of interest. This approach has been successfully applied to a single dataset of a paper-based wet friction clutch obtained from an Accelerated Lifetime Test (ALT) on the SAE#2 setup which will be described in Section 3. The result shows that the proposed relevant features exhibit clear trends.

The objectives of this work are twofold. Firstly, to validate the consistency of the previous result \(^{(5)}\) with more lifetime datasets obtained with different test setups and operational conditions. Secondly, to demonstrate the possibility to apply the pre-filtered HTLS, a variant of HTLS \(^{(6)}\) described above, for clutch monitoring. This variant actually performs the same procedure as proposed in \(^{(5)}\), but the pre-filtering process is integrated with the parameter estimation in a more elegant and compact way.

The structure of the paper is as follows. Section 1 discusses the motivation and the objectives of the paper. In Section 2, the pre-filtered HTLS is briefly reviewed. The experimental aspects, such as the SAE#2 test-setups and the procedures are discussed in Section 3. Results and discussion are presented in Section 4. Finally, some conclusions drawn are given in Section 5.

2. Feature extraction

Generally, there are two steps applied for the feature extraction in this work. The first step consists of pre-processing the signal prior to quantification. Second, quantification of the signal parameters using pre-filtered HTLS is made.

2.1. Pre-processing raw torsional vibration signals

Prior to the extraction of the proposed features, \(N\) uniformly sampled data points \(y_n, n = 0,1,\ldots,N-1\), are captured from a torsional vibration signal, measured in the post-lockup phase. Capturing the data points starts from the time instant when the relative velocity between the drum and hub approaches zero (lock-up time), until the time record length
is equivalent to \( N \) data points. Afterwards, the data points are centred around zero and scaled by the standard deviation according to the following equation:

\[
s_n = \frac{y_n - \bar{y}}{\sigma} \tag{1}
\]

where \( s_n \) is the normalised version of \( y_n \), \( \bar{y} \) and \( \sigma \) are respectively the arithmetic mean and the standard deviation of \( y_n \).

2.2. Pre-filtered HTLS method

Pre-filtered HTLS is a variant of the HTLS method. In the following sub-sections, HTLS and the variant are briefly reviewed.

2.2.1. A brief review of the HTLS method

In principle, this method assumes that a transient signal with constant signal parameters, uniformly sampled \( y_n, n = 0,1,\ldots,N-1 \), can be modelled as a sum of \( K \) exponentially damped sinusoids, where \( K \) is the model order:

\[
y_n = \sum_{k=1}^{K} c_k \exp[-\alpha_k n T_s + j(2\pi f_{dk} n T_s)] + e_n
\]

\[
= \sum_{k=1}^{K} c_k z_k^n + e_n
\tag{2}
\]

where \( j = \sqrt{-1} \), \( c_k \) is the \( k \)th complex amplitude, \( \alpha_k \) is the \( k \)th decaying factor, \( f_{dk} \) is the \( k \)th damped natural frequency, \( T_s \) is the sampling period, \( z_k = \exp[-(\alpha_k + j2\pi f_{dk}) T_s] \) is the \( k \)th signal pole, and \( e_n \) is the complex noise. The detailed explanation concerning the method is fully given in [7].

The signal parameters \( c_k, \alpha_k, \) and \( f_{dk} \) are computed using the following algorithm:

**Given** \( N \) uniformly sampled data \( y_n, n = 0,1,\ldots,N-1 \), and a model-order \( K \). The data points \( y_n \)’s are mapped into an \( L \times M \) Hankel matrix \( Y \) as follows

\[
Y = \begin{bmatrix}
y_0 & y_1 & \cdots & y_{M-1} \\
y_1 & y_2 & \cdots & \cdot \\
\vdots & \vdots & \ddots & \vdots \\
y_{L-1} & \cdot & \cdots & y_{N-1}
\end{bmatrix} \tag{3}
\]

with \( L > K, M > K, \) and \( L + M - 1 = N \). It is recommended that the Hankel matrix \( Y \) should be chosen as square as possible in order to achieve the best accuracy. In practice, one can choose \( M \) from \([2N/5] \) to \([2N/3] \) where \([\cdot]\) denotes a ceiling operator.

**Step 1.** Compute a truncated Singular Value Decomposition (SVD) of \( Y \):

\[
Y = U_{L \times K} \Sigma_{K \times K} V_{M \times K}^H \tag{4}
\]

**Step 2.** Compute the Total Least Squares (TLS) solution \( Z_U \) or \( Z_V \) of the inconsistent and over-determined shift-invariant equation of \( U \) or \( V \):

\[
U^\dagger \approx U \downarrow Z_U
\]

\[
V^\dagger \approx V \downarrow Z_V^H \tag{5}
\]
The up (down) arrow on the $U$ and $V$ matrices means deleting the top (bottom) row. The superscript $H$ on the matrices above denotes Hermitian conjugate. Moreover, matrices $Z_U$ and $Z_V$ consist of the estimates signal poles $z_k$. Once the signal poles $z_k$ are obtained from $Z_U$ or $Z_V$, the estimated signal parameters $\alpha_k$ and $f_{dk}$ can then be extracted.

**Step 3.** Compute the complex amplitude $c_k$ by solving a set of over-determined equation in Least Squares (LS) sense

\[
\begin{bmatrix}
1 & \ldots & 1 \\
z_1 & \ldots & z_K \\
z_1^2 & \ldots & z_K^2 \\
\vdots & \ddots & \vdots \\
z_1^{N-1} & \ldots & z_K^{N-1}
\end{bmatrix}
\begin{bmatrix}
c_1 \\
c_2 \\
\vdots \\
c_K
\end{bmatrix}
= 
\begin{bmatrix}
y_0 \\
y_1 \\
y_2 \\
\vdots \\
y_{N-1}
\end{bmatrix}
\]

\[\text{.......................................................... (6)}\]

### 2.2.2. Pre-filtered HTLS method

It is well known that linear filtering process in the time-domain using either a Finite Impulse Response (FIR) filter or an Infinite Impulse Response (IIR) filter constitutes convolution between input signal and impulse response of the filter. The convolution operation can be simplified by a means of a matrix multiplication between a filter matrix $H \in \mathbb{C}^{p \times p}$ and an input signal vector $y \in \mathbb{C}^{p \times 1}$, such that $y_f = Hy \in \mathbb{C}^{p \times 1}$ corresponds to filtered signal vector. If the input and filtered signals are row vectors instead of column vectors, the filtering process becomes $y_f^T = y^T H^T \in \mathbb{C}^{1 \times p}$.

As was reported in (8), as the filtered signal can be possibly distorted by using an IIR filter by the presence of artefact components due to the poles of the IIR filter, only a FIR filter is therefore considered in this work. Let the FIR filter $\hat{h}_n$ have length $q$, and the filter be windowed by a function $w_n$ in order to avoid Gibbs phenomenon due to signal truncation. Furthermore, to eliminate the end effect due to the filtering, the windowed impulse response of the FIR filter $h_n = \hat{h}_n w_n$, $n = 1, 2, \ldots, q$ which also represents a set of filter coefficients, is mapped into a Toeplitz matrix $H_{\text{FIR}}$ with zero-padding as follows (6)

\[
H_{\text{FIR}} = 
\begin{bmatrix}
h_{q} & \ldots & h_1 & 0 & \ldots & \ldots & 0 \\
\vdots & \ddots & & \ddots & \ddots & \ddots & \vdots \\
h_q & \ldots & h_1 & \ldots & \ldots & \ldots & 0 \\
0 & \ldots & h_2 & \ldots & \ldots & \ldots & \vdots \\
\vdots & \ddots & & \ddots & \ddots & \ddots & \vdots \\
0 & \ldots & 0 & h_q & \ldots & h_1 \\
\end{bmatrix}
\in \mathbb{C}^{p \times p}
\]

\[\text{.......................................................... (7)}\]

\[
\tau = \left[ \frac{q+1}{2} \right]
\]

In order to obtain the filter coefficients for constructing the Toeplitz filter matrix as expressed in Eq. 7, the FIR filter has to be designed. The filter can be designed based on the window design method (9). The window function used in this work is the Kaiser
window with the parameter $\beta = 5.658$ as recommended in (10). In this work, a Matlab toolbox was used for the FIR filter design.

The algorithm of the pre-filtered HTLS is as follows:

Given $N$ uniformly sampled data $y_n$, $n = 0, 1, \ldots, N - 1$, is mapped into an $L \times M$ Hankel matrix $\mathbf{Y}$ as in (3), a model-order $K$, and a windowed impulse response of band-pass FIR filter ($h_n$, $n = 1, 2, \ldots, q$) encompassing the frequency range of interest.

**Step 1.** Construct an FIR filter matrix $\mathbf{H} = \mathbf{H}_{\text{FIR}}$ as in (7), where $\mathbf{H} \in \mathbb{C}^{p \times p}$. For the filtering via right (respectively left) multiplication $p = M$ and $\mathbf{H}_r = \mathbf{H}^T$ (respectively $p = L$ and $\mathbf{H}_l = \mathbf{H}$) are taken.

**Step 2.** Pre-filtered $\mathbf{Y}$ by matrix multiplication

$$\mathbf{Y} = \mathbf{YH}_r \quad \text{(alternatively} \quad \mathbf{Y} = \mathbf{H}_l \mathbf{Y}) \quad \text{.......................................................... (8)}$$

**Step 3.** Apply the HTLS algorithm to $\mathbf{Y}$ to obtain the signal parameters $c_k, \alpha_k$, and $f_{dk}$.

Finally, once the signal parameters $\alpha_k$, and $f_{dk}$ are obtained, the undamped natural frequency $f_{nk}$ and the damping ratio $\zeta_k$ can be computed as follows

$$f_{nk} = \frac{1}{2\pi} \sqrt{(2\pi f_{dk})^2 + \alpha_k^2} \quad \text{.......................................................... (9)}$$

$$\zeta_k = \frac{\alpha_k}{2\pi f_{nk}} \quad \text{.......................................................... (10)}$$

### 3. Accelerated Lifetime Test (ALT)

As was stated in the first section, features can be considered to be relevant if they are effectively related to the degradation and show clear trends during the lifetime. To validate whether the proposed features are relevant or not for the clutch monitoring, as a consequence, lifetime data are absolutely needed. In this work, the concept of ALT is applied in order to obtain the lifetime data in a reasonable period. ALT’s were carried out on two SAE#2 test setups as will be discussed in the following sub-sections.

#### 3.1. SAE#2 test setups

In principle, two SAE#2 test setups used in this work are similar wherein both drum and hub are free to rotate. The fundamental difference between both setups is only the position of input and output motors. Figure 2 shows a complete scheme of the first test setup. Since the instrumentations of both test setups are similar, only the driveline of the second test setup therefore is shown in Figure 3. As has been seen in both figures, in the first setup, the axes of the motors are in line of the clutch axis, and in the second setup, the axes of the motors are in parallel of the clutch axis. The first test setup as depicted in Figure 2 basically consists of three main systems: driveline, control, and measurement system. Furthermore, the driveline consists of nine components: AC motor for driving the input shaft (1), input velocity sensor (2), input flywheel/drum (3), multi-disc clutch package (4), torque sensor (5), output flywheel/hub (6), output velocity sensor, AC motor for driving output shaft/hub (8), and a hydraulic system (11-20). Control systems (22) are used for both controlling the input oil pressure to the clutch and for the velocity
of input- and output-shafts. A multi-channel measurement system (22) is used to measure all relevant dynamic signals.

**Figure 2** A scheme of the first SAE#2 test setup

**Figure 3.** A top view scheme of the driveline of the second SAE#2 test setup

### 3.2. Test description

Two ALT scenarios were proposed for the first SAE#2 test setup. There were 4 ALT’s with different types of friction materials carried out in the first scenario using an ATF-A under the same operational conditions. In the second scenario, 4 ALT’s with identical types of friction materials were carried out using an ATF-B under different operational conditions. Furthermore, there was only one scenario proposed for ALT’s on the second setup, wherein 4 ALT’s with identical type of friction materials were performed using an ATF-C under different operational conditions.

### 3.3. Description of a duty cycle

Initially, while both flywheels are rotating at the same speeds in opposite direction, the motors are power-off and the pressurised oil is simultaneously applied into the clutch package at time $t_f$. The pressurised oil actuates the clutch piston, pushing the friction and separator discs towards each other. The latter occurs between time $t_f$ and $t_c$ which is
called the filling phase. Consecutively, contact is established between separator and friction discs. As a result, the transmitted torque starts to increase at time $t_e$. This is known as engagement phase wherein the relative speed between the drum and hub decreases and dissipated heat results in an increase of the ATF temperature. Finally, the clutch is completely engaged when the relative speed reaches zero at lockup time $t_s$. Typical signals recorded during a representative duty cycle of each test setup are shown in Figure 4.

![Figure 4](image.png)

**Figure 4** Representative recorded signals of one duty cycle. Left and right panels correspond to the first and second test setups respectively

### 4. Results and discussion

Torsional vibration signals from a torque sensor are used in this work to extract the features. As was previously stated, the torque sensor is probably difficult to install in the real-life application. However, to apply the features in the real-life clutch monitoring, one can use different kind of sensors, such as a high resolution optical encoder or a Ferraris sensor, to measure the torsional vibration signals.

The representative torsional vibration signal and its TFR’s obtained from the ALT’s on the first and second test setups are given in Figure 5 – Figure 7. The left and right panels respectively show the normalised time-domain signal and its TFR. The figures generally show that the frequency component(s) at relatively high torsional mode(s) is varying as indicated by the arrows. However, it can also be seen that the frequency components at low torsional vibration modes are relatively constant.
Figure 5. A representative TFR of the torsional vibration signal obtained in the first scenario of the first setup

Figure 6. A representative TFR of the torsional vibration signal obtained in the second scenario of the first setup

Figure 7. A representative TFR of the torsional vibration signal obtained on the second setup
Prior to the extraction of the proposed features (dominant undamped natural frequency $f_t$ and the corresponding damping ratio $\zeta_t$) using the pre-filtered HTLS method, a proper band-pass FIR filter must be designed. The frequency bandwidth of the filter is determined based on a visual inspection on the TFR of the signal in such a way, so that the proposed features certainly lie in the frequency range of interest during the service-life for identical ALT’s. Therefore, the same band-pass FIR filter can be applied for the identical ALT’s. Since there is only one frequency component in the range of interest, the selected model order $K$ is 2.

Based on the TFR’s depicted in Figure 5 – Figure 7, one can see that the approximate dominant natural frequency of each test scenario on the first and second setup are respectively 8, 10, and 22 Hz. Furthermore, based on the location of the dominant frequencies, the bandwidth frequency of the FIR filter corresponding to each test scenario on the first and second setup could be respectively determined as 5 – 20 Hz, 5 – 20 Hz, and 15 – 40 Hz.

The extracted features in function of the number of duty cycles $N_{cycles}$ of each test scenario on the first and second setup are plotted in Figure 8 – Figure 10. For readability reasons, the features are scaled by dividing the features by the feature values obtained from the first measurement which represents the beginning of service-life ($\hat{f}_t = f_t/f_1^i$ and $\hat{\zeta}_t = \zeta_t/\zeta_1^i$), where $\hat{f}_t$, $\hat{\zeta}_t$ and $f_1^i$, $\zeta_1^i$ are respectively the scaled features, and the features obtained from the first measurement. In general, the figures show that the extracted features deviate from their initial values during the service-life indicating the progress of the degradation.

**Figure 8.** The normalised features $\hat{f}_1$’s and $\hat{\zeta}_1$’s in function of successive duty cycles ($N_{cycles}$) of the first test scenario on the first setup
In the early service-life, it can be seen in Figure 8 – Figure 10 that the \( \hat{f}_t \)'s decrease. According to Eq. 10, as expected, the \( \hat{\zeta}_t \)'s decrease as shown in Figure 8 and Figure 10. However, different trends of the \( \hat{\zeta}_t \)'s are observed in Figure 9. This could be explained by the fact that the dominant frequency is strongly coupled with another frequency component, as can be seen in Figure 6, which possibly leads to an inaccurate estimation of \( \hat{\zeta}_t \). The characteristic shown in the early service-life can be interpreted as a premature degradation.

After the early stage, the features \( \hat{f}_t \)'s and \( \hat{\zeta}_t \)'s remain relatively stable which indicate a constant failure rate, as can be observed in Figure 8 – Figure 10. This also indicates that the clutches are still in the useful service-life and far from the failure.

In further stage, as can be particularly seen in Figure 8 and the first data in Figure 9, the features change. The particular changes perhaps indicate that the clutches approach the end of their lifetime. Note that the ALT’s were not carried out until the end of the lifetime. Further research is obviously required to draw more definitive conclusions concerning the feature trends until the end of the lifetime.
5. Conclusions

A pre-filtered HTLS method has been applied to extract the proposed features from several data sets of paper-based wet clutches. The extracted features show some trends during the service-life which might lead to conclusions about the progress of the degradation. The trends generally show similar characteristic which confirm the one obtained in the previous study \(^5\), especially the trends in the early stage and in the useful service-life stage. However, further research is needed to apply the pre-filtered HTLS for condition monitoring of wet clutches until the end of the lifetime.

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