Parameter Sensitivity and Measurement Error Propagation in Torque Estimation Algorithms for Induction Machines

C. Bastiaensen¹, W. Deprez¹, W. Symens², J. Driesen¹

¹Department of Electrical Engineering, University of Leuven, Kasteelpark Arenberg, 10 – 3001 Leuven, Belgium
Phone: +32 16 32 10 20, Fax: +39 16 32 19 85, E-mail: cindy.bastiaensen@esat.kuleuven.be.

²Flanders' Mechatronics Technology Center Celestijnenlaan, 300 D – 3001 Leuven, Belgium
Phone: +32 16 32 80 50, Fax: +32 16 32 80 64, E-mail: wim.symens@fmtc.be.

Abstract – This paper studies error propagation and parameter sensitivity based on a torque estimation model for induction machines. The model is based on the equation describing the interaction of rotor flux linkage and rotor currents. Contrary to classical schemes for induction motor control this is an open loop scheme, however, the model still requires different machine parameters. Therefore the parameters sensitivity of the model is performed. For validation, the model is implemented in the real-time environment dSPACE and a test induction machine is subjected to different combinations of speed and torque profiles. The identified model can be used to replace mechanical torque measurement devices or as a backup of a cheap torque sensor.

Keywords – Parameter Sensitivity, Torque Estimation, Induction Machines, Measurement Error Propagation, Drives, Open Loop Schemes.

I. INTRODUCTION

Sound and reliable torque measurement equipment is expensive; but often also calibration and maintenance are required. In some applications, due to a variety of reasons, the overhead of torque measurement devices is unwanted or not optimal and even sometimes there is no space to equip the application with such systems. In most cases it is sufficient to make an accurate estimation of the torque. In the literature, many expressions for the electromechanical torque of an induction machine can be found [1]–[6]. Based on such basic formulae requiring mostly only electrical quantities, sometimes also the mechanical speed, the mechanical torque can be estimated. In this paper, it is illustrated that such torque estimation expressions could be used to replace mechanical torque measurement devices. Moreover, the result of the estimation model can be used as a backup for a less accurate, but cheap, torque sensor. The model used in this paper is based on the equation describing the interaction of rotor flux linkages and rotor currents; so contrary to classical schemes for induction motor control this is an open loop scheme. However, as in all observers, control schemes or drives for induction machines, different (machine) parameters are required [1]–[6],[8]–[9]. Therefore, the parameter sensitivity of the used torque estimation model is studied. The determination of the required motor parameters is based on the classical T-equivalent circuit and the corresponding no-load and locked-rotor tests [8]–[11]. There is also a proposal for a new IEC standard for determining the T-equivalent circuit parameters. It was found that there is a potential inadequacy in this proposal. Namely, the calculations require the rotor bar height, for which the standard proposes to use a certain fraction of the frame size in millimeters. It is shown that this is often erroneous, it leads to incorrect parameters and thus faulty torque estimates.

Section two describes the developed torque estimation algorithm. The model requires motor parameters, how these parameters are determined, is elucidated in section three. The model validation is performed in the first part of section four. The second part of this section studies the parameter sensitivity. Finally, the fifth section of this paper describes how the propagation of measurement errors through the model influences the accuracy of the estimated torque.

II. TORQUE ESTIMATION MODEL

There are several expressions that describe the developed electromechanical torque of an induction machine [1]–[6]. However, they all originate from the same general description of three-phase ac machines, namely the stator and rotor voltage equations with \( R_s \) and \( R_r \) representing the resistance of the stator and the resistance of the rotor windings (stator referred)
respectively:

\[
\begin{align*}
(u_{sa}) &= R_s (i_{sa}) + \frac{d}{dt} (\psi_{sa}) \\
(u_{sb}) &= R_s (i_{sb}) + \frac{d}{dt} (\psi_{sb}) \\
(u_{sc}) &= R_s (i_{sc}) + \frac{d}{dt} (\psi_{sc})
\end{align*}
\]

with \(i_{ra}, i_{rb}\) and \(i_{rc}\) the phase currents in the rotor, \(i_{sa}, i_{sb}\) and \(i_{sc}\) the phase currents in the stator, \(u_{ra}, u_{rb}\) and \(u_{rc}\) the rotor phase voltages, \(u_{sa}, u_{sb}\) and \(u_{sc}\) the stator phase voltages, \(\psi_{ra}, \psi_{rb}\) and \(\psi_{rc}\) the rotor flux linkage and \(\psi_{sa}, \psi_{sb}\) and \(\psi_{sc}\) the stator flux linkage. In case of squirrel-cage induction machines, the phase voltages of the rotor are zero. These equations can be simplified according to quite established transformations in electrical engineering [1], namely the transformation of rotor and stator variables to the \(a/b/\beta\)-reference frame (Clarke transformation) [12] and the transformation to the \(d/q\)-reference frame (Park transformation) [13]. Depending on the chosen reference frame, this can lead to different expressions for the torque [1]–[6]. The expression for torque, obtained according to a reference frame rotating with the rotor flux, becomes:

\[
T_{cl} = p(\psi_{rq} i_{rd} - \psi_{rd} i_{rq}) = \frac{L_h}{L_r} \psi_{rd} i_q = \frac{L_r^2}{L_r} i_\mu i_q
\]

with \(i_\mu\) the magnetizing current, \(p\) the number of pole pairs, \(L_h\) the magnetizing inductance and \(L_r\) the rotor leakage inductance. The torque estimation model, build in Simulink®, is based on this expression; see Figure 1. Note that this is an open loop scheme although there is an internal feedback. The input for the model consists of two currents \(i_a\) and \(i_b\) and the mechanical rotor speed. The block ‘fase _ rotorflux’ performs the Clarke and Park transformations. The block ‘angle _ rotor’ integrates the mechanical speed in order to obtain the rotor angle in radians. Equation (3) is concretized by the block ‘torque’ and in ‘fluxmodel’ the angular velocity of the rotor flux and the magnetizing current are calculated from the rotor equations:

\[
\begin{align*}
\frac{L_r}{R_r} \frac{di_\mu}{dt} + i_\mu &= i_d \\
\omega_\mu &= \omega_r + \frac{R_r}{L_r} i_q
\end{align*}
\]

III. PARAMETER DETERMINATION

As can be noticed from equations (3) to (5); the torque estimation model requires three machine-parameters. In particular they are the magnetizing inductance \(L_h\), the rotor resistance \(R_r\) and the rotor leakage inductance \(L_r\) from the single phase T-equivalent circuit representation of induction machines. The elements of this equivalent scheme can be obtained by basic, straightforward calculations based on a short circuit and a no load test. However, in order to obtain correct torque estimates, skin effect in the rotor bars and, to a minor extent, temperature corrections should be taken into account. There are several ways to consider this skin effect [14], [15]. Anyhow, owing to the nature of this effect, the size (and shape) of the conductors is of importance, therefore, at least the height of the rotor bars should be known in order to be able to correct the rotor impedance [14], [15]. Such a correction is also provided in the proposal for a new IEC standard for the determination of the quantities of equivalent circuit diagrams. In a first proposal the rotor bar height was estimated as being 20% of the motor frame size in millimeters, but the idea that the height of the rotor bar is just a fixed fraction of the machine frame is not reasonable. Therefore a new proposal has been made, namely the rotor bar height is estimated as:

\[
h = (0.21 - \frac{2p}{100}) \ast \text{motor frame size}
\]

This is an understandable practice since in this way, still no manufacturer data is required, however, in this application it is too inaccurate. In the aspiration of obtaining precise measurements the design data are necessary to bring into account the skin effect. Table 1 gives an overview of the errors made by using one of the proposals. The first column shows the used formula for the rotor bar height, the second column shows the corresponding value for \(h\), the third column shows the calculated torque and the fourth column in terms of percentage the deviation of the torque.

<table>
<thead>
<tr>
<th>formula (h)</th>
<th>(h [\text{m}])</th>
<th>(T_c [\text{Nm}])</th>
<th>% (T_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 * motor frame size</td>
<td>0.0264</td>
<td>23.2705</td>
<td>95.0592</td>
</tr>
<tr>
<td>(0.21 - \frac{2p}{100}) * motor frame size</td>
<td>0.0251</td>
<td>23.5665</td>
<td>96.2684</td>
</tr>
<tr>
<td>design data</td>
<td>0.0219</td>
<td>24.1827</td>
<td>98.7855</td>
</tr>
</tbody>
</table>

Table 1: Rotor bar height
In the standard determination of the parameters also temperature corrections can taken into account, however 25°C is used as reference temperature. A small deviations of this reference temperature will hardly not influence the determined motor parameters and therefore the environment parameters can be neglected in the torque estimation model.

IV. VALIDATION AND PARAMETER SENSITIVITY

A. Model verification

Once a set of parameters is determined for a particular induction machine, the torque estimation model can be implemented in a real-time environment so that the torque can be estimated on-line. The model was validated by using the real-time tool dSPACE and also by a in-house DSP based measurement and rapid prototyping platform [16]. For the validation, a test-machine was subjected to different combinations of speed and torque profiles. The currents are measured and read in, also the speed is measured by an encoder, but it could also be estimated or measured by other means. For the case where an encoder is used, the angular position can be directly determined and so the block 'angle_rotor' can be omitted. The torque is measured by a torque transducer.

In Figure 2, the result of such a validation experiment is depicted. The machine under test for this particular case was an off-the-shelf 7.5 kW premium efficiency two pole induction machine with a rated speed of 2925 rpm. The rated current is 13.7 Amps and the rated torque is 24.5 Nm.

To the machine under test, a sinusoidally changing load at low frequency was applied. The speed is changed step-wise, but during each period of this sine it is kept constant. In Figure 2, only part of the complete experiment is plotted. It is the part where there is a speed reversal. The speed is depicted in the subplot above, the measured torque (blue) and the estimated torque (red) are plotted in the subplot below. There is quite some noise on both torque signals, but nevertheless it can be found that the estimate perfectly coincides with the measurements. Other test profiles confirmed these findings, though they also revealed some problems at very low speeds. This will be investigated in the future, however, since this is not relevant for this paper it will not be discussed.

B. Parameter sensitivity

Now, the sensitivity of the torque estimation model to inaccuracy or changes of the different parameters is investigated. In this analysis of the parameters, the influence of the different parameters on the estimated torque is simulated, first by changing in turn one of the parameters and secondly by holding in turn one of the parameters. The first set of simulations is carried out for rated conditions. From the first approach, varying only one parameter, it can be concluded that the order of the parameters according to the sensitivity on the calculated torque is: \( R_r > L_h > L_r \). Figures 3, 4 and 5 show the results of the second approach. On the vertical axis, the calculated torque is plotted in Nm, whereas on the two axes composing the horizontal plane the deviations of the respective parameters is plotted in percentage of their rated values. The conclusion from these figures is that \( L_h \) and \( L_r \) are opposing parameters, that \( L_h \) and \( R_r \) strengthen each other and that the influence of \( L_r \) is negligible compared with the influence of \( R_r \). In this context, it should be mentioned that the measurement accuracy of the tests described in Section 3, is very important for the accuracy of the parameters determination. For this paper, the measurement accuracy was assessed based on the 'per point' residual of the least squares fitting of the supplementary losses. The supplementary losses are calculated according to [11].

Up till now, the susceptibility of the model for parameter inaccuracy was only studied for rated conditions. However, since the model should equally perform for non-rated conditions, also simulations for parameter sensitivity are performed at different speeds. Table 2 gives an overview of the sensitivity.
of the motor parameters as function of speed. The first column shows the used percentage of the speed, the second, third and fourth columns show in terms of percentage the deviation of the torque for a deviation of respectively $+10\% L_r$, $+10\% L_h$ and $-10\% R_r$. The last column shows the combination of the third and fourth column. Only this combination is made with a view to the expression for torque (1).

<table>
<thead>
<tr>
<th>% n</th>
<th>$L_r$ +10%</th>
<th>$L_h$ +10%</th>
<th>$R_r$ -10%</th>
<th>$L_h$ +10% $R_r$ -10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>99.5092</td>
<td>101.6708</td>
<td>91.1918</td>
<td>92.4912</td>
</tr>
<tr>
<td>50</td>
<td>99.6141</td>
<td>105.2902</td>
<td>94.8525</td>
<td>99.1733</td>
</tr>
<tr>
<td>25</td>
<td>99.7884</td>
<td>111.8955</td>
<td>101.6168</td>
<td>112.6709</td>
</tr>
<tr>
<td>10</td>
<td>99.9407</td>
<td>118.3588</td>
<td>108.3422</td>
<td>127.8129</td>
</tr>
<tr>
<td>5</td>
<td>99.9748</td>
<td>119.9067</td>
<td>109.9688</td>
<td>131.7707</td>
</tr>
<tr>
<td>1</td>
<td>99.9868</td>
<td>120.4588</td>
<td>110.5505</td>
<td>133.2165</td>
</tr>
</tbody>
</table>

Table 2: Sensitivity of the parameters as a function of speed

From these simulations some important conclusions can be drawn. Firstly, the influence of an inaccuracy on $L_r$ on the estimated torque is negligible and even decreases with the speed. For $L_h$ and $R_r$ the sensitivity is not negligible. An overestimation of $L_h$ makes that the torque will be overestimated. The overestimation becomes more serious with decreasing speed. It is not shown in the table, but evidently the torque is underestimated for a negative deviation of this parameter. The behavior for an over- or underestimation of $R_r$ is more complicated. This is manifested in the last two columns of table 2. When the deviation on $R_r$ is negative, the torque is underestimated at rated speed but there is a transition and at low speeds the torque is overestimated.

From the table it is clear that errors or deviations on the parameters of such a model can lead to serious over- or underestimations of the torque especially at low speeds. In fact, at low speed, the errors are amplified. A simultaneous deviation of 10% on two parameters can lead to an overestimation of 33% at low speed.

V. PROPAGATION OF MEASUREMENT ERRORS

In order to assess the influence of measurement errors on the behavior of the model an analogous approach as in the previous section is chosen. The influence of deviations on these inputs is calculated. This is necessary to check if the model does not amplify the measurement errors as they propagate through it. The model requires two currents and the mechanical rotor speed as input. The rated values for mechanical speed and current are taken as a reference; for the currents this also means that the amplitudes of $i_a$ and $i_b$ are considered to be 120 electrical degrees apart. Figure 5 depicts the influence of possible combinations of current measurement errors of up to ten percent of their rated values. For these calculations the phase angle between the currents and the speed was kept constant. This figure reveals a linear behavior of the model on current measurement errors; as far as amplitude is concerned. But more important, it clearly indicates the importance of the correct allocation of the model inputs as the influence of errors on $i_b$ is stronger than those on $i_a$. This is because of the choices made when setting up the Clarke and Park transformations. Another important remark is that an additional effect on the estimated torque occurs when the amplitude values of $i_a$ and $i_b$ are not varied in the same way. The fact is, the calculated torque oscillates with a frequency of 100 Hz (with 50 Hz being the rated electrical frequency). The amplitude of the torque oscillation depends on the amplitude difference of $i_a$ and $i_b$, this is not visible in figure 5 since there the rms value for the calculated torque is plotted. A similar oscillation occurs when the phase shift between $i_a$ and $i_b$ is not exactly 120 electrical degrees. The amplitude of the 100 Hz torque oscillation varies linearly with the phase shift. Finally, figure 6 represents the suscep-
tibility of the model to speed measurement errors. Addi-
tionally, it should be mentioned that the above does not apply for
measurement and numerical noise. Partly due to the presence
of integrators in the model, this noise is forwarded and even
slightly amplified by the model. However, since the currents
and speed signals could be filtered appropriately, this effect is
of minor importance.

![Figure 6. Influence of current inaccuracy on the torque estimation](image)

![Figure 7. Influence of speed inaccuracy on the torque estimation](image)

VI. CONCLUSIONS

The susceptibility of a torque estimation model to parame-
ter deviations and measurement errors was studied. The model
was discussed briefly and it was illustrated that, in general, it
gives a good estimation of the actual torque. However, several
simulations revealed that, especially at low speeds, the sensi-
tivity of the model for parameter faults is not negligible.

motor drives with adaptive system model,” *International conference on
Power electronics, machines and drives*, Bath, UK, April 2002, pp. 498–
503.
Electric Vehicle,” *IEEE Trans on Energy Conversion* 18, 2003, No. 1,
pp. 1–10
tion Motor Drives for Low Speed Operation,” *IEEE Trans on Power
Electronics* 19, 2004, No. 6, pp. 1608-1613
and d-q transformation for induction machine,” Engineering Appl. Of
Artificial Intelligence 18, 2005, No. 1, pp. 57–63
servers for Induction Motors,” *IEEE Trans on Industry Applications* 39,
2003, No. 4, pp. 1127–1135
ters From Manufacturer Data,” *IEEE Trans on Energy Conversion* 19,
2004, No. 2, pp. 310–317
on Magnetics* 38,2002, No. 4, pp. 1774–1779
[10] IEEE, *Standard Test Procedure for Polyphase Induction Motors and
Generators*, IEEE Std 112-1996, New York/USA, IEEE Power Engi-
neering Society 1996
proach,” Oxford University Press, New York/USA, 1992
AIEE*, 1929
ulation of Induction Machines,” *Industry Applications Society Annual
Meeting*, October 1989, Conference Record 1-5, pp.38–44 Vol.1
with Skin-Effect,” *The Pacific Journal of Science and Technology* 5,
2004, No. 2
[15] C. Gherasim, J. Van den Keybus, “DSP implementation of power mea-
surements according to the IEEE trial-use standard 1459,” *IEEE Transac-
tion on instrumentation and measurement* 53, 2004, no.4, pp. 1086-1092